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**METEORIC MATTER IN
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by V. G. Fesenkov

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By V. G. Fesenkov

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TABLE OF CONTENTS

<u>CHAPTER</u>	<u>PAGE NO.</u>
ABSTRACT	1
CHAPTER 1	
Data on Matter in Interplanetary Space	2
CHAPTER 2	
Light Phenomena Representing Possible Streams of Hyperbolic Meteors	15
CHAPTER 3	
Integral Effect of the Decay of Periodic Comets	28
CHAPTER 4	
The Probability for the Capture of Comets as a Function of Their Orbital Inclination	45
CHAPTER 5	
Possible Number and Total Surface of Asteroids	50
CHAPTER 6	
Effect of Asteroid Disintegration	56
CHAPTER 7	
Motion of Cosmic Dust in Interplanetary Space	80
CHAPTER 8	
Density Determination in the Medium of Interplanetary Meteoric Matter on the Basis of Stationary Conditions ..	103
CHAPTER 9	
Theoretical Determination of Spatial Density of Meteoric Matter in Interplanetary Space	109
CHAPTER 10	
The Exchange Rate of Zodiacal Light Matter	122

<u>CHAPTER</u>	<u>PAGE NO.</u>
CHAPTER 11	
Zodiacal Light According to Visual Observations and Considerations on Its Photometric Study	128
CHAPTER 12	
Theory of Night Sky Airglow	132
CHAPTER 13	
Determination of the Effective Height of Ionosphere Airglow	170
CHAPTER 14	
Separation of the Components Comprising the Nocturnal Sky Luminosity	175
CHAPTER 15	
Photometric Observations of Zodiacal Light and Their Interpretation	205
REFERENCES	221

METEORIC MATTER IN INTERPLANETARY SPACE

V. G. Fesenkov

ABSTRACT

The theoretical and observational data concerning meteors and cosmic dust which give rise to zodiacal light are reviewed. It is shown that this matter must have its origin primarily within the solar system, because the sun cannot capture interstellar matter. The effect of hyperbolic meteors is investigated in detail, and it is shown that their effect on zodiacal light is negligible.

The author attempts to derive the distribution of meteors within the solar system. It is assumed that they have been captured by Jupiter and have disintegrated over several million years. It is found that most cosmic matter, consisting of disintegration products of objects captured by Jupiter, must be located in the orbital plane of Jupiter. It is shown that approximately the same total mass of asteroidal matter lies within each stellar magnitude. The reasons for assuming that the solar system continuously accumulates a dust-like substance are discussed.

The motion of a dust particle in space, subject to the sun's attraction and light pressure, is studied. The time required for a particle to fall into the sun from any distance is computed. Due to the photoeffect, the particle will accumulate a charge, which is found not to influence its motion.

Stationary distribution of meteoric matter in space is derived, based on the previously-discussed mechanisms of production and decay. The density distribution is inversely proportional to the distance from the sun.

The authors investigate the distribution of dust outside the ecliptic, and expressions for the zodiacal light isophotes are derived.

The nature of the earth's emissive layer is investigated, as well as the light-scattering properties of the troposphere. The problem of separating the galactic and ionosphere components from observations of the sky is discussed.

CHAPTER 1

Data on Matter in Interplanetary Space

Cosmic space abounds in matter occurring in gaseous and dust-like states. Clouds of cosmic dust are primarily concentrated in the galactic plane. Some of them can be sharply distinguished on the light background of the Milky Way, producing considerable absorption and also scattering of light - which is more significant for small wavelengths. /3*

This scattering of light by the galactic dust-like matter produces the so-called galactic luminescence, which amounts to about 80% of the total stellar brightness toward the center of the galaxy, and 30 - 60% in other directions (Ref. 1, 2). Similar matter also occupies our planetary system, accumulating primarily in the ecliptic plane. Absorption in this material is barely noticeable, and its scattering is also very small. Its scattering can be noticed on the dark background of the night sky only due to its proximity to the bright sun.

The energy distribution in the spectrum of this luminescence does not differ at all from the spectrum of the sun itself (Ref. 3, 4, 5), as must be the case for particles which are large as compared with the length of a light wave.

The conclusion can readily be drawn that separate particles of this dust medium, entering the earth's atmosphere, can be observed in the form of shooting stars.

We have not as yet made any assumptions with respect to the structure of this medium. It can be stated that this medium consists of individual meteor streams, representing the disintegration product of comets or consisting of scattered dust clouds, and continuously encompasses interplanetary space. /4

In every case, each particle must describe an elliptical orbit around the sun, independently from the others, if only the solar repulsive force is not too great and does not cause such a particle to escape in a rolling, spiral-like manner into interstellar space. We are constantly encountering this phenomenon in the cosmic tails of the second type. Sometimes explosions occur in the comet's nuclei, and clouds of fine dust are scattered into space; they are picked up by light pressure, enter into the comet tail, and then leave the solar system. In every case, the particle dimensions of the interplanetary dust medium have a lower limit which is caused by the presence of solar light pressure and which is not,

* Note: Numbers in the margin indicate pagination in the original foreign text.

naturally, dependent on distance. This limit is determined by the condition that a change in the impulse of the light pressure equals the acceleration of gravity, i.e.,

$$\frac{\pi \rho^2 E_0 r_0^2}{cr^2} = \frac{k^2 M_0 \cdot \frac{4}{3} \pi r^3 \delta}{r^2},$$

where E_0 is the solar constant at the distance r_0 , c - velocity of light, ρ , δ , r - radius, density of the particle, and its distance from the sun. Assuming that the particle density is 1, we find a limiting radius for it of approximately 1μ .

Such particles cannot be a constant component of the dust medium in the solar system. If, in actuality, the smallest particles have a radius on the order of 10μ , then their mass will be on the order of 10^{-8} grams. According to the determination made by Epik (Ref. 6), a meteor of the apparent magnitude m - penetrating the earth's atmosphere at a mean velocity of 55 km/sec - has the mass

$$M = 2.512^{-m} \cdot 0.25 \text{ grams}$$

This corresponds to the following meteor dimensions of different apparent magnitudes m (density is assumed to be 3):

/5

m	0	2	4	6	8	10	12	13	14
ρ (mm)	2.7	1.47	0.79	0.43	0.23	0.13	0.068	0.050	0.037

Meteors which are produced by particles having extremely small dimensions (10μ) must belong approximately to the 18th stellar magnitude.

However, this limit must be considerably increased, in view of the fact that particles having a dimension of several microns cannot exist for a long period of time in the solar system. Due to light deceleration, they must rapidly fall into the sun. Actually, in the simplest case of circular motion the following expression can be readily derived for a change in the momentum during the time dt :

$$d(mv) = \frac{\alpha \pi \rho^2 E_0 r_0^2}{cr^2} dt,$$

where $\alpha = \frac{v}{c}$ is the constant of annual aberration.

Thus, the following expression can be derived for a change in the orbit radius as a result of radiative deceleration:

$$\frac{dr}{r} = \frac{3}{2} \frac{E_0}{c^2 \rho} \frac{r_0^2}{r^2} \frac{dt}{\delta}.$$

Thus, the time a particle takes to reach the sun is

$$t = \frac{c^2 \rho \delta r^2 \cdot 10^{-7}}{9 E_0 r_0^2} \quad \text{years}$$

If, for example, $\rho = 10\mu = 10^{-2}$ mm, $\delta = 1$, $r = r_0 = 1.5 \cdot 10^{13}$ cm, then $t = 7 \cdot 10^3$ years - a quantity which is negligible as compared with the amount of time the solar system has existed (Ref. 7, 8). /6

A mass of dust-like matter can therefore exist for a long period of time, under the condition that the particles having the same dimension are constantly replaced by new ones. Due to the fact that they exist for a longer period of time, the larger particles will tend to dominate. Therefore, it is difficult to establish the upper limit of meteorite dimensions from theoretical considerations.

Let us turn to observational data.

Statistics on telescopic meteors have produced some interesting information. It has been found that the number of meteors increases with each subsequent stellar magnitude, so that their total mass remains approximately the same and equals 11 kilograms/day (Ref. 9) for the entire terrestrial surface. After two billion years, this forms a layer on the earth which is somewhat less than 1 cm thick.

This increase in the number of meteors has been ascertained to the 10th stellar magnitude. Some researchers have expressed the opinion (Watson) that the limiting magnitude of meteors which still exist does not exceed the 13th stellar magnitude. Particles having similar dimensions can exist in the solar system for a comparatively short period of time - on the order of hundreds of thousands of years - if only they rotate around the sun along a closed orbit. In every case, almost all particles in interplanetary dust mediums, with the exception of the very small ones, can be observed in the form of telescopic meteors, using present-day observational methods.

However, it must be stated that these meteors actually belong to the solar system, and not to galactic space.

This can be proved in two different ways, namely: by investigating the velocity of individual meteors, on the one hand, and by studying the photometric properties of all of them as a group, on the other hand. Up until recently, the opinion prevailed that the majority of meteors move at hyperbolic velocities with respect to the sun (Hoffmeister, Nissl', Epik, and others [Ref. 10]). However, all of these conclusions were based on visual observations, which /7

were interpreted on the basis of arbitrary assumptions. When absolutely objective photographic material was obtained, it was found that all of the specific velocities corresponded to elliptical orbits.

A careful analysis of British observations of meteorites, which was recently carried out by Porter (Ref. 11), showed that only in 10% of the cases can one suspect hyperbolic velocities, and they are problematical. In every case, an investigation of the velocities of meteors and bolides did not reveal the presence of hyperbolic orbits. This does not mean that there are absolutely no such orbits, but this does mean that they do not play a significant role in any case in the total balance of the dust medium in the solar system. The so-called Taurid cosmic stream clearly exemplifies the incorrectness of recent conclusions regarding meteor velocities.

On the basis of visual observations, Hoffmeister pointed out the existence of a stream of cosmic meteors which are carried through the solar system from dark nebulae in the Taurus constellation toward the Scorpion constellation.

However, when a camera equipped with a device for determining velocity was used at Harvard to photograph fourteen meteors belonging to this stream, they were all found to have a period of rotation around the sun of 3.3 years. According to a study by Uippl (Ref. 12), they are related to the comet Enke which rotates around the sun along a smaller orbit, and they have no relationship to the hyperbolic trajectories which were assumed initially.

The photometric method is based on the following considerations. Let us assume that a stream of meteors, which moves initially along rectilinear and parallel trajectories, enters the solar system. Each meteor describes a hyperbola around the sun; the greater the curvature of the hyperbola, the closer it approaches the sun.

/8

It is possible to determine the amount of matter which passes through a unit volume of space at each given moment - i.e., the density of the dust medium which is exclusively determined by hyperbolic orbits of cosmic meteors.

If this density is determined in relative units, it is possible to calculate the brightness distribution of the dust medium along both sides of the sun, and to compare it with observations. In the case of the hypothetical stream of Hoffmeister, it is found that the nature of the brightness distribution along both sides of the sun is approximately the same, but the brightness itself differs sharply at the same angular distances (Ref. 13). For example, if we assume that cosmic meteors have a rather large hyperbolic velocity, when the semimajor axis $a = -0.1$, we obtain the following for the brightness of the eastern and western branches of zodiacal light in the same

units at different angular distances l from the sun:

	$l = 30^\circ$	$l = 60^\circ$	$l = 90^\circ$
Western Branch	8.5	3.9	2.7
Eastern Branch	4.4	1.8	1.2

At a lower velocity, the difference between both branches will be even larger. However, observations have shown that there is no significant difference between the branches of zodiacal light. This shows that there is no significant asymmetry in the spatial distribution of matter. Thus, galactic streams of meteors, which can exist only in a small number - if not singly - are either not present, or do not play a significant role in each case.

Thus, the interplanetary medium consists of meteors which move almost exclusively along elliptical orbits. It is important to know the distribution of these particles in terms of dimensions.

/9

It has already been noted that, according to observations, the total mass of meteors at each interval of stellar magnitude is approximately the same.

Let $A(m)$ be the number of shooting stars in the brightness interval which is expressed in stellar magnitudes from $m - \frac{1}{2}$ to $m + \frac{1}{2}$; let $A(m)dm$ be the number of shooting stars in the interval from $m - \frac{1}{2}dm$ to $m + \frac{1}{2}dm$. Let $M(m)$ be the mass of a shooting star whose brightness is m .

According to convention

$$A(m) M(m) = C = \text{const.}$$

Since the mass is proportional to the brightness - other conditions being equal - we have

$$M = M_0 \cdot 2.512^{-m},$$

where M_0 is the mass corresponding to $m = 0$.

Therefore, we have

$$\rho = \rho_0 \cdot 2.512^{-\frac{m}{3}}$$

and, consequently

$$d\rho = -\frac{1}{3} \rho_0 2.512^{-\frac{m}{3}} (\ln 2.512) dm.$$

We find:

$$A(m) dm = \Phi(\rho) d\rho = \frac{C_0^3 3 \lg e}{M_0 0.4 \rho^4} d\rho \sim \frac{d\rho}{\rho^4}.$$

Thus, the distribution function $\Phi(\rho)$ of the number of meteors with respect to dimensions is proportional to ρ^{-4} . The number of meteors rapidly increases with a decrease in their dimensions. The deviation from this law is insignificant up to the 10th stellar magnitude.

There is no doubt that very small particles predominate in the dust medium, but these particles cannot be long-lived. The dust medium in the solar system exists for a short period of time. It must either be trapped from interstellar space, or be generated within the solar system in some way. /10

Many authors have expressed the opinion, and continue to express this opinion, that the sun - when passing through any cosmic cloud - can capture part of it. Thus, for example, Bobrovnikov - when attempting to explain the small lifetime of comets on the basis of the capture theory - assumed that the sun captured part of the meteoric matter from the region in space occupied by the Orion nebula approximately a million years ago.

A significant portion of the cosmogonical hypotheses - advanced by Hoyle, Veytsekker, Schmidt - assume or substantiate such a capture.

There is just one capture mechanism which has been known up to the present; this is the mechanism of large perturbations. The passing sun captures matter from interstellar space with the aid of an additional planet body. By means of its attraction, this planet can decrease the kinetic energy of the passing body and can make it a satellite of the sun for a certain period of time. This mechanism was studied in detail by Tisserand, Kallandro, G. N'yuton, and others (Ref. 14). Tisserand (Ref. 15) set forth his well-known criterion for the identity of a body before or after large perturbations:

$$\frac{1}{a} + 2\sqrt{p} \cos i = C,$$

where a is the semimajor axis; p - parameter; i - inclination; C - constant in the Jacobi interval.

However, this process is unlikely. It is assumed that a foreign body not only passes close to the sun, but also falls within the sphere of influence of a planet - for example, Jupiter - in a definite manner. It must be kept in mind that the body can again pass close to Jupiter after a certain number of rotations; after undergoing large perturbations, it can be ejected from the solar system.

This mechanism can play the role of changing long-period comets into short-period comets. A family of comets can thus be formed which have planetary orbits. However, this is not of practical importance for bodies having a galactic origin. This mechanism cannot play a significant role in the transformation of cosmic meteor orbits.

In its rotation around the galactic center, can the sun capture cosmic matter under any circumstances and cause it to rotate along an elliptical orbit, the dimensions of which are rather small as compared with the distance from the galactic center?

The distance to the galactic center is about 40,000 light years. The distance from the earth to the sun in the same units equals $8 \cdot 10^{-5}$. This is the time required by light to traverse the distance between the sun and the earth. Thus, the distance from the sun to the earth represents $0.4 \cdot 10^{-9}$ of the distance to the galactic center.

Let us examine a mass which is concentrated in the galactic center, the sun which rotates around it along a circular orbit, and a third body having insignificant mass.

It can be shown (Ref. 16) that under these conditions the orbital elements of this body, with respect to the sun, satisfy the same criterion of Tisserand, only written in a somewhat different form:

$$\frac{1}{a} + 2\sqrt{p} \cos i = C - C_0,$$

The difference of the Jacobi constants $C - C_0$ is given in the right part; C characterizes the orbit of the body, and C_0 characterizes the orbit of the sun itself. Apparently, the most favorable conditions for capture occur when the body moves along the same circular orbit as /12 the sun, but only slightly inclined. Thus, the convergence of both bodies occurs with the least relative velocity. Since $C = C_0$ in this case, we have:

$$\frac{1}{a} + 2\sqrt{p} \cos i = 0,$$

while

$$p = a(1 - e^2), \cos i \simeq 1.$$

As can be seen, the semimajor axis for the orbit of the body with respect to the sun cannot under any conditions be small. As a rule, it is found that $a < 0$.

Thus, it is absolutely impossible for the sun to capture interstellar matter.

It can thus be shown that meteors of a planet system having elliptical-type orbits cannot be captured by the galactic medium.

In the same way, planets from the solar system cannot surround themselves with meteoric matter during any perturbations in the movement of counter meteoric streams.

Observations point exactly to this. For example, comets have repeatedly passed around Jupiter at a very small distance from it, and have undergone strong perturbations. However, we do not know of one comet which rotates around Jupiter.

Meteoric satellites of the earth are equally impossible. The only possible way for the earth to capture meteors is to decrease their kinetic energy as a result of resistance produced by high atmospheric layers. Under specific approach conditions, a meteor passes at a very high altitude in the earth's atmosphere, where the density is so small that it can penetrate the atmosphere completely, only decreasing its kinetic energy. As a result, the meteor describes an elliptical orbit around the earth, and by means of rotation again returns to the previous atmospheric layer.

Disturbances from the moon can change the orbit slightly, and can make the meteor a satellite of the earth after a certain number of rotations (Ref. 17).

/13

Thus, meteoric matter must belong to the solar system itself, and in all probability is continuously produced in it. All of this matter, scattering the solar light, produces the phenomenon which is known as zodiacal light. This phenomenon was known to the Egyptians in far antiquity, who drew it on their obelisks. It has the form of a lens extending along the elliptical plane, in the center of which the sun is located. The brightness of the zodiacal light increases rapidly toward the sun, and also toward the elliptical plane. The phenomenon of zodiacal light is therefore represented in the form of light cones of the east or west, depending on the position of the ecliptic with respect to the horizon. Both cones are combined, in addition, by a continuous zodiacal band which is very weak and indefinite, extending along the ecliptic. At the side opposite the sun, this band significantly intensifies the brightness in the form of an oval, turbid spot - the so-called counter glow. The earth must be immersed in this matter, and therefore zodiacal light is propagated in actuality along the entire sky, somewhat increasing the luminosity of the night sky (Ref. 18).

Let us now turn to the problem of interpreting the observed brightness distribution in the zodiacal light. The separation of the actual brightness of the zodiacal light itself from the total brightness of the sky, which is primarily caused by the luminosity of high ionosphere layers, is a difficult problem, but one which can be solved.

Let $\mu f(\vartheta)$ be the portion of scattered solar light produced by a unit volume of dust medium at the distance r from the sun at the angle ϑ , between the incident and scattered rays. If $\Phi(\rho)d\rho$ represents the distribution function of particles with respect to the dimension ρ and $\mu(\rho)f(\vartheta, \rho)$ characterizes scattering produced by one particle (ρ), which is calculated per unit of solid angle, then - assuming the distribution law of densities in the form r^{-n} , where n is the indefinite exponent - we find the following expression for the observed brightness of zodiacal light in the ecliptic plane (Ref. 19):

/14

$$j = J \frac{r_0}{\sin^{n+1} l} \int_{\rho_1}^{\infty} \mu(\rho) \Phi(\rho) d\rho \int_0^{\pi} f(\vartheta, \rho) \sin^n \vartheta d\vartheta,$$

where l is the angular scattering from the sun; J - illumination from the sun; j - illumination from the zodiacal light.

Since the particles are large as compared with λ , it can be assumed that the scattering indicatrix $f(\vartheta, \rho)$ does not depend on ρ .

Therefore, we have

$$= J \frac{r_0}{\sin^{n+1} l} \int_{\rho_1}^{\infty} \mu(\rho) \Phi(\rho) d\rho \int_0^{\pi} f(\vartheta) \sin^n \vartheta d\vartheta.$$

It can be verified that the result depends slightly on the nature of the function $f(\vartheta)$. For very small particles, comparable with the length of a light wave, $f(\vartheta)$ has a very asymmetric form in the propagation direction of an incident ray. For significant dimensions, $f(\vartheta)$ depends on the particle form. Diffraction increases the scattering first; the presence of surface shading decreases the scattering first. On the average, we can assume that $f(\vartheta) = \text{const} = 1$.

Utilizing the recurrence formula

$$\int_0^{\pi} \sin^n \vartheta d\vartheta = - \frac{\sin^{n-1} \vartheta \cos \vartheta}{n} \Big|_0^{\pi} + \frac{n-1}{n} \int_0^{\pi} \sin^{n-2} \vartheta d\vartheta,$$

we can calculate the second integral for any whole n .

The best agreement with observations is obtained for $n = 1$. We thus have:

$$j = J \frac{r_0}{\sin^2 l} (1 - \cos l) \int_{\rho_1}^{\rho_2} \mu(\rho) \Phi(\rho) d\rho.$$

Drawing a comparison with observations which I carried out in Kitaba in 1934 (Ref. 20), we have the following agreement between observed and calculated brightnesses according to the given formula.

z	30°	40°	50°	60°	90°
Observed	7.62	5.05	3.20	2.10	1.00
Calculated	7.45	4.28	2.81	2.00	1.00

This means that the density of matter in the ecliptic plane changes according to the law r^{-1} . There is actually no need to search for a more exact law.

Other scattering indicatrices having a more complex form give approximately the same results.

The following hypothesis can be formulated with respect to the origin of meteors in the solar system.

Meteors are produced as the result of a gradual disintegration of asteroids which primarily occupy the area between the orbits of Mars and Jupiter. The disintegration can arise both due to collisions with cosmic meteors, which are passing through the solar system at a great velocity, due to collisions with meteors which belong to the solar system, and due to collisions between asteroids. The total mass of asteroids is small and may possibly not exceed 0.1 of the earth's mass. However, their total surface in all probability greatly exceeds the surface of the earth, due to the extreme fractionation of asteroids. If, for example, the earth is divided by equal bodies having a radius which is ten times smaller, then the total surface of these bodies will be ten times greater than the earth's surface. Consequently, the surface sum of bodies having a radius of 6 km each, which are obtained by dividing the earth, will be one thousand times greater than the terrestrial surface. A body having a radius of 6 km represents a rather typical asteroid which can be observed in modern /16 telescopes.

There is no doubt that an enormous majority of the asteroids have small dimensions. In actuality, it is impossible to give any real limit for their radius. They can have a radius of several meters, even centimeters, down to the size of grains. Thus, the enormous surface which the asteroidal ring represents is a primary obstacle to the flight of cosmic meteors through the solar system. The fine dust which is produced during these collisions scatters in all directions, and fills space. In this connection, the following problem can be formulated and solved.

Let a planet-asteroid move along a circular orbit at the distance R from the sun. For a certain reason, particles of dust are emitted from this planet uniformly in all directions, with a definite relative

velocity. Let us assume that this process occurred a long time ago, so that a stationary density distribution has been established. It is necessary to determine a relative amount of matter at any point in space. As the result of lengthy calculations, as will be shown below, the following density distribution law is obtained in the plane of the asteroidal circle, for different values of the emission velocity v_0 and for different distances r from the sun.

TABLE 1

r	$v_0^2 = 1$	2	3	4	r^{-1}
0.2	2.50	2.55	1.20	0.63	2.00
0.4	1.00	1.00	1.00	1.00	1.00
0.8	0.43	0.46	0.31	0.54	0.50
1.0	0.57	∞	∞	∞	0.40
1.2	0.32	—	0.25	1.24	—
1.6	0.09	—	0.052	0.41	—

The orbit radius of the asteroid and its velocity are assumed ^{/17} to equal unity. If it is assumed that the radius $R = 4$ astronomical units, then - according to the third law of Kepler - we have

$$v^2 R = \text{const}, \quad v = \frac{\text{const}}{\sqrt{R}},$$

i.e., the orbital velocity, assumed to be unity, represents approximately 15 km/sec. The earth is located at a distance of 0.25 assumed units from the sun. It can be seen that in the region of space around the earth's orbit the density of matter changes in close accordance with the law r^{-1} , at particle emission velocities of $v_0 = 1 - \sqrt{2}$. For large velocities, the density increase towards the sun is slowed down, and then completely stops; close to the sun, there is a vacuum.

In a manner similar to this, the vacuum close to the sun occurs for small emission velocities. Consequently, it can be concluded that the required density distribution is obtained during the mechanism under consideration only for significant emission velocities confined within rather narrow limits. However, it is interesting to note that for all possible velocities, either large or small, the asteroidal ring itself is always a region of increased density for the dust-like matter. It thus definitely follows that a mass of fine dust must be accumulated in the circle of asteroids; as can be imagined, this explains the phenomenon of the zodiacal band. However, there is reason to doubt the fact that the particle emission velocities can be rather large.

We should point out that the results given above were obtained

without taking the radiative deceleration into account. The necessity of large emission velocities disappears with such deceleration. In addition, let particles continuously emerge from the asteroidal zone, just as from a reservoir, and let them gradually approach the sun as the result of radiative deceleration. Let this process continue for a rather long period of time - in fact, for the entire lifetime of the solar system. Let us assume that the distribution of matter is in a stationary state. In this case, the amount of matter, located in a volume of space which is traversed per unit of time, must be everywhere the same - i.e.,

/18

$$\omega r^2 \frac{dr}{dt} f(r) = \text{const},$$

where ω is a solid angle; $f(r)$ is the density of matter in space.

Substituting the value of $\frac{dr}{dt}$ which was given above, we find that

$$f(r) = \frac{\text{const}}{r}.$$

Thus, the phenomenon of radiative deceleration leads to the density distribution law r^{-1} , in exact agreement with observations.

Assuming that $\mu(\rho) = \frac{A}{4} \rho^2$, where A is the particle albedo, we find:

$$j = J \frac{K r_0}{\sin^3 l} (1 + \cos l) \frac{A}{4 \rho_1}.$$

If it is assumed that the brightness of the zodiacal light is equivalent at a distance of $l = 60^\circ$ to one star of the 5th magnitude per square degree, then we have

$$\frac{j}{J} = \left(\frac{180}{\pi} \right)^2 \cdot 2.512^{-(26.5+5)} = 8.2 \cdot 10^{-10}.$$

For purposes of definition, let us assume that $A = 0.4$. The mass of matter, per unit volume of space at the distance $r = 1$, will be:

$$M = K \int_{\rho_1}^{\rho_2} \Phi(\rho) d\rho \frac{4\pi}{3} \rho^3$$

or

$$M = \frac{4}{3} \pi \delta K (\ln \rho_2 - \ln \rho_1).$$

Assuming $m = 13$ for the limiting stellar magnitude of the meteor, i.e.,

/19

$$\rho_1 = 5 \cdot 10^{-3} \text{ cm},$$

we find that

$$K = 10^{-24}$$

and therefore

$$M = 6 \cdot 10^{-3} \text{ degrees} -$$

the mass per 1 cm^3 at the distance from the earth to the sun.

The total mass in a region of space having a radius equalling the radius of the earth's orbit is

$$1.3 \cdot 10^{18} \text{ grams,}$$

assuming that it has spherical symmetry according to the law of density change r^{-1} . This amounts to $2.6 \cdot 10^{-10}$ of the earth's mass.

This represents the preliminary data on zodiacal matter, which have been derived from our previous studies.

CHAPTER 2

Light Phenomena Representing Possible Streams of Hyperbolic Meteors

As was indicated above, there is no basis for assuming that cosmic meteors - penetrating the solar system at hyperbolic velocities from interstellar space - benefit the interplanetary dust medium in any significant way. In every case, such meteors must belong to individual streams located primarily in the galactic plane. According to Hoffmeister, the most pronounced stream of this type is related to the region occupied by the dark nebulae in the Taurus constellation, which is located close to the ecliptic plane. Hoffmeister's conclusions cannot be regarded as correct at the present time, since this stream has a comparatively small heliocentric velocity. The opinion that there are hyperbolic meteoric streams can be shown to be untenable from the photometric aspect. /20

Let us examine the following general problem. Let a meteoric stream, which has a uniform structure and is not limited in any way, pass through the solar system. Located at a great distance from the sun, each particle of the stream moves along parallel trajectories at the same velocity v_0 . The attraction from the sun disturbs the structure of this stream. It is necessary to determine the density of matter at any point in space, at a distance from the sun which is not particularly large, and then to evaluate the light phenomena for the observer on the earth. /21

Let us assume that the center of the sun is the origin, and a straight line which is parallel to the initial stream velocity v_0 is the x-axis. Let the initial coordinates of the meteoric particle be $x_0 \rightarrow \infty$ and y_0 . On the basis of the kinetic energy integral

$$v^2 = k^2 \left(\frac{2}{r} - \frac{1}{a} \right) \quad (1)$$

we find the velocity v at any distance r from the sun. The expression

$$v_0^2 = -\frac{k^2}{a} \quad (2)$$

determines the actual semiaxis of the hyperbola a . The equation for the particle trajectory in polar coordinates (r, ψ) is

$$r = \frac{-a \operatorname{tg}^2 \psi}{1 + \cos \vartheta + \operatorname{tg} \psi \sin \vartheta}, \quad (3)$$

where ψ is the angle which the line of apsides forms with the x-axis.

In order to determine the angle ψ , which is apparently related to the eccentricity, we shall employ the area integral which can be written in the form

$$y_0 v_0 = q v_q$$

as applied to the orbit perihelion, which is located at the distance q from the sun. Utilizing (1) and (3), we find:

$$y_0 v_0 = -a (\sec \psi - 1) k \sqrt{\frac{2}{-a (\sec \psi - 1)} - \frac{1}{a}},$$

from which it follows

$$y_0^2 v_0^2 = -a k^2 (\sec^2 \psi - 1),$$

and finally

$$\sec \psi = \sqrt{1 + \frac{y_0^2}{a^2}}$$

or

$$y_0 = -a \operatorname{tg} \psi. \quad (4)$$

Finally, the angle (n, r) between the normal to the trajectory /22 at the point (r, ϑ) and the radius-vector is determined by the relationship

$$\operatorname{tg}(n, r) = -\frac{dr}{r d\vartheta},$$

from which we find

$$\cos(n, r) = \frac{1}{\sqrt{1 + r^2 \frac{\sec^2 \psi}{p^2} \sin^2(\psi - \vartheta)}}. \quad (5)$$

The parameter p has the value

$$p = -a(e^2 - 1) = -a \operatorname{tg}^2 \psi.$$

Let us now assume that the elementary rectangle $\delta x_0 \delta y_0$ is occupied by material particles which move independently of each other under the influence of the sun's attraction. In a certain time interval, this rectangle is deformed by the area $\delta \sigma$. We can find the relationship

$$B = \frac{\delta \sigma}{\delta x_0 \delta y_0},$$

which shows the expansion ratio of a plane element and, consequently, the reduction ratio of the surface density in the x, y plane, as compared with the initial value.

If δt is the time interval in which a particle traverses the segment δx_0 , then we have

$$\delta x_0 = v_0 \delta t.$$

Apparently, within an accuracy of a small quantity of a high order, we have

$$\delta\sigma = v\delta t\delta n,$$

where δn is the distance between contiguous trajectories, taken along the normal. Therefore, we have

$$B = \frac{v\delta n}{v_0\delta y_0}$$

or

$$B = \frac{v\delta r \cos(n, r)}{v_0\delta y_0}.$$

But on the basis of (3), we have

/23

$$-\frac{a\delta r}{r} = \operatorname{cosec}^2\psi [2(1+\cos\vartheta)\operatorname{ctg}\psi + \sin\vartheta]\delta\psi.$$

On the other hand, from (4) we have

$$\delta y_0 = -a \sec^2\psi \delta\psi.$$

After various transformations, we obtain:

$$B = \sqrt{\frac{2}{r} - \frac{1}{a}} \cdot \frac{a\sqrt{-a}r^2}{y_0^3} \left[\frac{y_0}{a} \sin\vartheta - 2(1+\cos\vartheta) \right] \times \\ \times \frac{1}{\sqrt{1 + \frac{r^2 a^3}{y_0^4} \left(\frac{y_0}{a} \cos\vartheta + \sin\vartheta \right)^2}}$$

or finally

$$B = \left[\frac{r^2}{y_0^2} \sin\vartheta - 2(1+\cos\vartheta) \frac{ar^2}{y_0^3} \right] \times \\ \times \frac{\sqrt{1 - \frac{2a}{r}}}{\sqrt{1 + \frac{r^2}{y_0^4} \left(y_0 \cos\vartheta + a \sin\vartheta \right)^2}} \quad (6)$$

Thus, B represents a function of polar coordinates r, ϑ , which is dependent on the parameters y_0 and $v_0 = \frac{k}{\sqrt{-a}}$. However, for each

given point with given r, ϑ , the magnitude of y_0 must depend on v_0 . Actually, describing the trajectory equation in the form

$$r = \frac{y_0^2}{-a(1+\cos\vartheta) + y_0 \sin\vartheta},$$

we obtain from it

$$y_0 = \frac{r \sin \vartheta}{2} + \sqrt{\frac{r^2 \sin^2 \vartheta}{4} - ar(1 + \cos \vartheta)}. \quad (7)$$

We place a positive sign before the radical, since it can always be assumed that $y_0 > 0$. /24

In order to change to the spatial density D , it is only necessary to take into account the symmetry with respect to the x -axis. The entire amount of matter which is included within the torus

$$2\pi y_0 \delta x_0 \delta y_0,$$

will also be included within the torus $2\pi y \delta \sigma$ during the same time interval. Consequently, we have

$$D = \frac{y_0}{y} \frac{\delta y_0 v_0}{\delta r v \cos(n, r)}$$

or

$$D = F(r, \vartheta, v_0) = \frac{y_0}{yB},$$

while

$$v = r \sin \vartheta.$$

In actuality, the fact must be taken into account that in the x, y plane the elementary area $\delta x_0 \delta y_0$, corresponding to $y_0 > 0$, can enter both the region of positive values of y , as well as the region of negative values of y . Let us use D_1 and D_2 to designate the density values obtained respectively for $y_0 y > 0$ and $y_0 y < 0$. In the final analysis, we have

$$D = D_1 + D_2 = \frac{1}{y} \left(\frac{y_0}{B} + \frac{y_0'}{B'} \right),$$

where B, y_0, B', y_0' correspond to the points r, ϑ , which are located symmetrically with respect to the x -axis.

In the case of very small ϑ , we have

$$y_0 = \sqrt{-2ar} + \frac{r\vartheta}{2}$$

and

$$B = \frac{\sqrt{1 - \frac{2a}{r}} r \vartheta}{\sqrt{1 - \frac{r}{2a}} (-2a + \vartheta \sqrt{-2ar})} - \frac{4ar^2}{(\sqrt{-2ar})^3 + 3(\sqrt{-2ar})^2 \frac{r\vartheta}{2}},$$

from which it follows that

$$B = \frac{\sqrt{-2ar} \vartheta}{-2a + \vartheta \sqrt{-2ar}} + \frac{4\sqrt{-2ar}}{2\sqrt{-2ar} + 3\vartheta}$$

or

$$B = 2 + \sqrt{-2ar} \vartheta \left(1 - \frac{3}{4a} \right)$$

Consequently, for $\vartheta = 0$ $B = 2$, no matter what r may be.

On the other hand, for arbitrary ϑ , but very small r , we have

approximately

$$y_0 = \sqrt{-a} \sqrt{r(1 + \cos \vartheta)}$$

and after substitution in (6) we again find $\lim B = 2$ for $r = 0$. In general, for

$$y_0 y > 0$$

the values of B are included within the limits $1 \leq B \leq 2$, and for

$$y_0 y < 0$$

within the limits of $2 \leq B < \infty$.

Let us apply these formulas to a specific case.

Let us use the radius of the earth's orbit as the unit of length, and a portion of the earth's rotation around the sun as the unit of time $\frac{1}{2\pi}$, which is equivalent to the assumption that $k^2 = 1$.

We shall have

$$v_0^2 = \frac{1}{a}$$

Let us assume that the velocities of hyperbolic meteors amounts to 100 km/sec, and above. In our units, this velocity equals

$$v_0^2 = \frac{100 \cdot 365,24 \cdot 24 \cdot 60 \cdot 60}{2 \cdot 3,14 \cdot 149\,500\,000} \approx 3,5.$$

The following corresponds to this:

$$a = -0,083.$$

We shall use the following in round numbers:

$$a = -0,1$$

and we shall calculate the values of B and D for different values of r and ϑ . Tables 2 and 3 present the results. Tables 4 and 5 present analogous values of B and D, calculated by V. A. Kurapov for $a = -1,0$. In the case $a = -0,1$, the values of D are shown in Figure 1.

/26

As can be seen from these Tables, the disturbing action of the sun is much more intense and includes incomparably larger areas of space when the moving particles have smaller kinetic energies.

Let us now calculate the brightness of the zodiacal light produced by matter belonging to such a meteoric stream. Let L represent the amount of light emitted by the sun per unit of solid angle; $\mu f(\vartheta)$ the scattering ability of matter (per one degree per one steradian); ξ , R - the distances of an element in space from the sun and from the earth.

The observed brightness of the zodiacal light will be

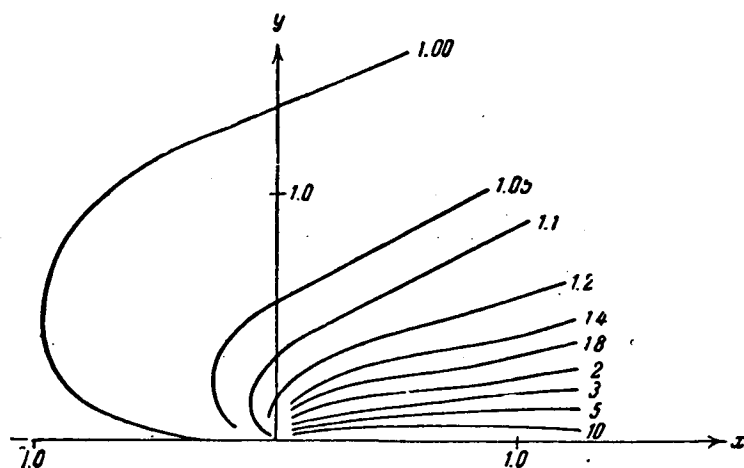


Figure 1
Lines of Equal Cosmic Matter Density, Produced by
Hypothetical Hyperbolic Meteors.

$$J = \mu L \int_0^{\infty} \frac{Df(\alpha) dR}{\xi^2}.$$

In the ecliptic plane

/30

$$\xi = r, \quad r = \frac{R \sin l}{\sin \alpha}$$

and therefore

$$\frac{JR}{L\mu} = \frac{1}{\sin l} \int_0^{\pi-l} Df(\alpha) d\alpha.$$

The quantity in the right part of the latter equation is proportional to the brightness of zodiacal light. We can estimate this quantity for $\alpha = -0.1$, and for a position of the earth on an orbit corresponding to $\vartheta = 60^\circ$. For different angular distances from the sun, we find:

	$l = 30^\circ$	$l = 60^\circ$	$l = 90^\circ$
Western branch of zodiacal light.....	8.5	3.9	2.7
Eastern branch of zodiacal light.....	4.4	1.83	1.18

It can be seen that the brightness distribution in both branches is approximately the same, in spite of the fact that the density structure is completely different in different directions.

/31

TABLE 2
VALUES OF B ($a = -0.1$)

/27

r		0.2	0.4	0.6	0.8	1.0
$\vartheta = \pm 2.9^\circ$. .	y	± 0.010	± 0.002	± 0.03	± 0.04	± 0.05
	y_0	0.205	0.293	0.361	0.420	0.472
	B	2.00	2.00	2.00	2.00	2.00
	y_0'	0.195	0.272	0.330	0.380	0.422
	B'	2.00	2.00	2.00	2.00	2.00
$\vartheta = \pm 15^\circ$. .	y	± 0.052	± 0.107	± 0.155	± 0.207	± 0.259
	y_0	0.226	0.342	0.424	0.510	0.590
	B	1.77	1.68	1.62	1.58	1.55
	y_0'	0.174	0.233	0.272	0.305	0.332
	B'	2.25	2.45	2.65	2.80	2.96
$\vartheta = \pm 30^\circ$. .	y	± 0.100	± 0.200	± 0.300	± 0.400	± 0.500
	y_0	0.250	0.399	0.516	0.635	0.748
	B	1.60	1.45	1.40	1.37	1.35
	y_0'	0.150	0.199	0.216	0.235	0.248
	B'	2.50	2.90	3.38	3.70	4.00
$\vartheta = \pm 60^\circ$. .	y	± 0.173	± 0.343	± 0.519	± 0.692	± 0.866
	y_0	0.280	0.473	0.657	0.836	1.014
	B	1.38	1.27	1.21	1.18	1.15
	y_0'	0.107	0.127	0.137	0.144	0.148
	B'	3.55	4.75	5.80	6.85	7.85
$\vartheta = \pm 90^\circ$. .	y	± 0.200	± 0.400	± 0.600	± 0.800	± 1.000
	y_0	0.273	0.483	0.687	0.890	1.092
	B	1.26	1.17	1.12	1.10	1.08
	y_0'	0.073	0.083	0.087	0.090	0.90
	B'	4.5	6.81	8.91	10.9	12.9
$\vartheta = \pm 120^\circ$. .	y	± 0.173	± 0.343	± 0.519	± 0.692	± 0.866
	y_0	0.219	0.394	0.580	0.748	0.920
	B	1.21	1.13	1.09	1.07	1.06
	y_0'	0.046	0.051	0.052	0.053	0.054
	B'	5.8	8.9	11.8	14.8	17.8
$\vartheta = \pm 150^\circ$. .	y	± 0.100	± 0.200	± 0.300	± 0.400	± 0.500
	y_0	0.221	0.345	0.470	0.590	0.694
	B	0.512	0.590	0.655	0.700	0.733
	—	—	—	—	—	—
	—	—	—	—	—	—
$\vartheta = \pm 177.1^\circ$. .	y	± 0.010	± 0.020	± 0.030	± 0.040	± 0.050
	y_0	0.012	0.022	0.032	0.042	0.052
	B	1.178	1.105	1.075	1.058	1.048
	—	—	—	—	—	—
	—	—	—	—	—	—

/28

TABLE 3

/28

VALUES OF D ($a = -0.1$)

$\delta \backslash r$	0.2	0.4	0.6	0.8	1.0	1.5
2.9	20.0	14.1	11.5	10.0	8.94	7.30
15°	3.99	2.79	2.34	2.08	1.90	1.63
30°	2.16	1.71	1.44	1.32	1.22	—
60°	1.35	1.16	1.09	1.03	1.04	—
90°	1.16	1.10	1.04	1.02	1.02	—
120°	1.09	1.03	1.03	1.02	1.01	—
150°	1.13	1.06	1.04	1.02	1.02	—
177°.1	1.02	1.00	1.00	1.00	1.00	—

On the other hand, in terms of absolute brightness both branches differ to a considerable extent from each other. There is no doubt that if the meteors have a smaller kinetic energy, this difference will be even greater. Thus, simultaneous observations on the brightness of both branches of zodiacal light provide a good means of determining the content of galactic matter in it.

/31

Such observations can be carried out only in December and in June, when both branches of the zodiacal light have equally favorable visibility conditions. In the first case, the ecliptic is located low over the southern horizon, and both zodiacal branches - which are located symmetrically with respect to the meridian and which pass into the weak zodiacal band at the side opposite the sun - are clearly visible at about midnight at low latitudes. The western branch is thus superimposed on the Milky Way, which makes observations much more difficult. Moreover, the phenomenon is generally very weak.

In June, or at the beginning of July at a latitude of about 45°, when the sun is located at a distance of about 21° above the horizon at midnight, and when there is absolutely no twilight, it is possible to see the so-called Northern Zodiacal Light, which combines both zodiacal branches toward the sun.

This phenomenon was apparently first observed by Campbell in the summer of 1907 at the Likskoy Observatory (Ref. 21) in the form of weak phosphorescence at the northern part of the horizon. In June, 1908, this phosphorescence was observed up to a distance of 18° above the horizon. As Fez has noted, the emission line 5577, which is usually observed in the spectrum of the sky, was not intensified in this phosphorescence. There was absolutely no twilight, since the sun had sunk below the horizon by approximately 30° during these observations.

On the basis of his observations at the Likskoy Observatory

TABLE 4
VALUES OF B ($a = -1.0$)

/29

	r	0.2	0.4	0.6	0.8	1.0	1.5
$\vartheta = 2^\circ$	y_0	0.637	0.905	1.111	1.285	1.439	1.77
	B	1.99	1.98	1.97	1.97	1.97	1.97
$\vartheta = \pm 15^\circ$	y_0	0.654	0.94	1.17	1.36	1.54	1.92
	B	1.91	1.88	1.86	1.84	1.82	1.69
	y'_0	0.60	0.84	1.01	1.16	1.28	1.54
	B'	2.08	2.10	2.13	2.16	2.17	2.23
$\vartheta = \pm 30^\circ$	y_0	0.663	0.969	1.22	1.44	1.64	2.08
	B	1.85	1.79	1.75	1.71	1.69	1.65
	y'_0	0.56	0.77	0.92	1.04	1.14	1.34
	B'	2.20	2.28	2.33	2.39	2.44	2.56
$\vartheta = \pm 60^\circ$	y_0	0.64	0.967	1.24	1.50	1.73	2.28
	B	1.74	1.64	1.59	1.54	1.50	1.44
	y'_0	0.47	0.62	0.72	0.80	0.87	0.98
	B'	2.39	2.59	2.72	2.85	3.08	3.35
$\vartheta = \pm 90^\circ$	y_0	0.558	0.863	1.131	1.38	1.62	2.19
	B	1.64	1.58	1.46	1.41	1.38	1.31
	y'_0	0.36	0.45	0.53	0.58	0.62	0.69
	B'	3.67	4.60	5.39	6.13	6.85	8.54
$\vartheta = \pm 120^\circ$	y_0	0.415	0.653	0.867	1.067	1.26	1.73
	B	1.58	1.47	1.39	1.35	1.32	1.25
$\vartheta = \pm 150^\circ$	y_0	0.221	0.352	0.471	0.583	0.693	0.966
	B	1.55	1.44	1.36	1.34	1.29	1.21
	y'_0	0.121	0.152	0.171	0.183	0.193	0.216
	B'	4.48	5.80	7.23	8.52	9.74	12.07
$\vartheta = \pm 177^\circ.1$	y_0	0.02005	0.0325	0.0439	0.0549	0.0656	0.0919
	B	1.42	1.33	1.27	1.23	1.21	1.16

on the northern zodiacal light, Fez attempted to determine the visible isophots by visual means, extrapolating them up to the intersection with the ecliptic. He discovered a lenticular form for the phenomenon which was rather symmetrical with respect to the sun, extending along the ecliptic at both of its sides by approximately 70° , and by 46° to the north of it. These observations did not carry very great weight, since they were not performed photometrically. Nevertheless, there is no doubt that zodiacal light is represented by approximately identical isophotes at both

/32

TABLE 5
VALUES OF D ($a = -1.0$)

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θ	0.2	0.4	0.6	0.8	1.0	1.5
2.°9	63.4	45.4	37.2	32.4	29.0	23.8
15°	12.22	8.61	7.15	6.14	5.55	4.71
30°	6.09	4.40	3.63	3.18	2.87	2.38
60°	3.26	2.39	2.01	1.83	1.65	1.44
90°	2.19	1.66	1.45	1.34	1.26	1.17
120°	1.92	1.48	1.33	1.23	1.17	1.11
150°	1.70	1.35	1.23	1.14	1.11	1.08
170°	1.41	1.21	1.14	1.11	1.08	1.05

sides of the sun. This excludes the idea regarding its connection with individual meteoric streams or with comets, as Elvey assumed (Ref. 22).

/32

Similar observations on the northern zodiacal light were carried out for several years by Barnard, S. Newcomb, and other American astronomers (Ref. 23, 24, 25). In particular, this phenomenon was observed in a systematic way by Newcomb in the mountainous regions of the United States and by Shveytsarya at an altitude of 7700 feet. He came to the conclusion that the zodiacal light was quite sharply restricted to the north of the ecliptic at a distance of 35° from the sun, much more sharply than the zodiacal cones or branches along the ecliptical plane.

"The limit of 35, which I have set nevertheless seems to me much more precise than any limit that has been, or can be, set in the plane of the ecliptic".

This statement by Newcomb is of great importance.

I personally carried out photometric observations on the northern zodiacal light in the vicinity of Talgar, 40 km east of Alma-Ata, at the beginning of July, 1946. The Lapot' mesa, on which the observatory was located, stands alone among the surrounding plain, rising 1400 m above sea level. The observations were performed with a radioactive photometer, and the brightness was expressed in absolute units by making a comparison with the extra-focal stellar images. It was quite difficult to reduce the observations, since as a rule they referred to large zenith distances, and required an exact knowledge of the atmospheric transmission index.

/33

The photometric data which I obtained did not enable me to

draw a conclusion as to the existence of asymmetry in the form of zodiacal light around both sides of the sun.

As was indicated above, generally speaking, both branches of the zodiacal light cannot be observed simultaneously. Moreover, photometric observations on this phenomenon are extremely scarce, and have not been conducted systematically. Therefore, one should not disregard the evidence of observers who have noted the extension of the zodiacal branches for several years under different conditions. There have been several such observations: Schmidt (1843 - 1855), Deshevrens (1875 - 1879), Geys, Weber (1849 - 1883), Jones (1854), Serpieri, and others. The summary presented by Schermann (Ref. 26) can be used to compare the extension along the ecliptic of both zodiacal branches on the basis of such observations. We should note, however, that the observations of Jones are not uniform, since he made them from the deck of a ship at different locations on the earth. The other observations can be compared with each other, if they were made by one and the same person with the same inclination of the ecliptic toward the horizon. On the average, it can be assumed that the difference in atmospheric conditions is excluded.

For purposes of illustrating the dependence of the degree of zodiacal light visibility on the inclination of the ecliptic, I constructed two graphs on the basis of observations by Honnorat (Ref. 27). For a period of several months, he observed the quality of atmospheric conditions, the brightness of the zodiacal light, and also its extension along the ecliptic in a certain arbitrary scale. Following is the graph which I constructed for December - when the ecliptic is slightly inclined toward the horizon - and February-March, when the observational conditions are much better and when the zodiacal light /34 can be traced much closer to the sun (1 designates the best atmospheric conditions and the greatest brightness) (Figure 2). As can be seen, in December zodiacal light is extremely weak for each atmospheric condition, but in February-March there is a definite correlation between the atmospheric transmission and brightness: a great amount of brightness, which generally reaches significant intensity, also corresponds to the best atmospheric conditions - i.e., to great atmospheric transmission. The zodiacal light, as well as its dimensions, significantly increases on high mountains, as was repeatedly observed by A. Humboldt (Ref. 28, 29). However, he assumed that the zodiacal light did not change in actuality, and that it depended entirely on atmospheric conditions. It is interesting to note that there is no such relationship for the total brightness of the night sky. On the contrary, there are indications that the brightness of the sky around the celestial pole increases with a decrease in the index of atmospheric transmission (Ref. 30).

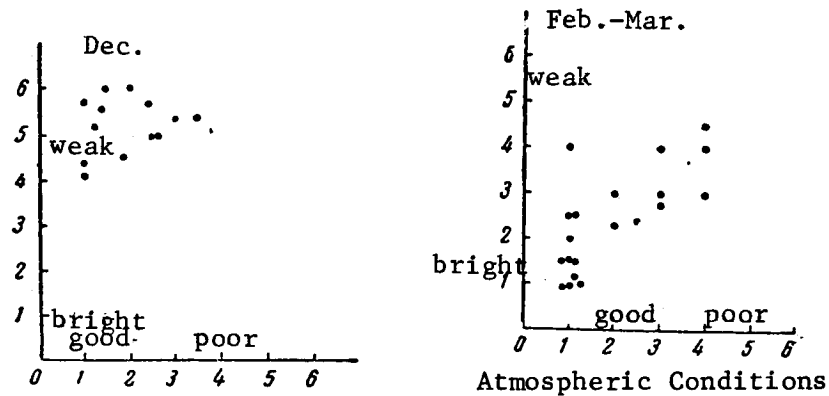


Figure 2

Brightness of the Zodiacal Light as a Function of Atmospheric Conditions.

The observed extension of a branch of the zodiacal light along the ecliptic can serve as a certain index of its absolute brightness. Comparing this extension for the morning branch in October and the evening branch in March, when visibility conditions are the same, on the average, for several years, we obtained the following results:

/35

Observer	Evening Branch (March)	Morning Branch (October)
Deshevrens	65°	65°
Geys, Weber (1849-1860).....	65°	70°
" " (1861-1871).....	77°	75°
" " (1872-1883).....	90°	104°

We should point out that at a distance from the sun of 90°, the visible brightness of the zodiacal light increases no more than 10-15% for an extension of 10°. Thus, it can be stated that the experimental observer with several years practice sees the evening and morning zodiacal light of the same extension and brightness.

Maunder also pointed out the equal brightness of both branches of zodiacal light; he observed this phenomenon in the tropics during his travels in India (Ref. 31).

This receives additional confirmation from the photometric

observations of Sandig, which he carried out in Equatorial Africa (Ref. 32) at the Vinduk Observatory ($\phi = -22^{\circ}.5$).

Thus, for example, for different isophotes, we have the morning and evening branches determined by Sandig on the average for June 29/30, and July 2/3 and 7/8, 1935:

Isophote	Evening	Morning	Isophote	Evening	Morning
1.0	35°	33°	2	51°	56°
1.5	44	46	3	77	82

The small difference does not exceed the deviations between individual days.

/36

It should be noted that these observations were carried out photographically, using a Leica with a lens aperture of 2.5 and exposures of 15^m - 30^m. The original negatives were so weak that it was necessary to make several reprints in order to intensify the contrast.

It is understood that the measurement accuracy cannot be great under these conditions.

Thus, it can be concluded that all of the data point to the absence of any influence from individual meteoric streams, in particular, hyperbolic streams, on the zodiacal light.

Streams of hyperbolic meteors, arriving from interstellar space, cannot be uniformly oriented in all directions in the ecliptic plane. One or two such streams can occur only by chance, which are captured in the solar system. The observed nature of zodiacal light contradicts such an assumption. The role of hyperbolic meteors must be insignificant.

CHAPTER 3

Integral Effect of the Decay of Periodic Comets

Let us examine the problem of the origin of meteoric matter in /37
the solar system.

Let us postulate the following mechanism for its gradual accumulation. We shall assume that comets having a parabolic velocity fall within the sphere of influence of the sun. A certain percent of the total volume of such comets will be captured by large planets, primarily Jupiter, and will remain within the planetary system. Gradually, under the dissolving influence of the sun, their mass separates into meteors and is distributed along the entire orbit in the form of a continuous circle.

The totality of meteoric streams, building up for several million years, comprises the general complex of cosmic matter, and the density distribution within it must be studied.

We have absolutely no information on the properties of individual orbits of meteoric streams. It is only possible to discuss the problem, because we assume that there is a very large number of several kinds of orbits. If this amount is so great that the law of large numbers can be applied, then we have a right to speak about the actual density distribution in the meteoric complex.

In the opposite case, we can only speak about the probable density distribution.

It must be noted that the following assumptions must be employed /38
in order to solve this problem:

1. The velocity of a parabolic comet outside the sphere of influence of the sun can be oriented in a completely arbitrary way. This is the only assumption which has a hypothetical character. I introduce it as the simplest assumption. However, nothing is changed or is rendered more complex if we assume any law for the orientation of comet velocities with respect to the solar system.

2. The orbit elements of a periodic comet do not depend on its mass.

3. The aggregate of cosmic dust does not have periodic density fluctuations.

The second assumption stands to reason; we must accept the third assumption, although it is not readily apparent. If periodic density changes occurred, then we could study only the mean density at a given point of space. Consequently, we must assume that in each volume of space, the amount of matter entering it equals the amount leaving it - i.e., that our complex of cosmic dust is everywhere in dynamic equilibrium.

Generally speaking, the elliptic orbit is characterized by five elements: i , Ω , π , a and e . In the given case, we have all possible values for π and Ω which can occur with equal probability, since the entire aggregate of matter has a circular symmetry.

In addition, we shall confine ourselves to investigating the density in the plane of symmetry, and therefore we shall assume $i = 0^\circ$. Consequently, we must study the total group of orbits with all possible a and e , which are located in one plane.

For purposes of simplicity, it is assumed that the planet Jupiter, which rotates around the sun in a circle with uniform velocity, is the only one to capture the comets. We shall try to determine the density distribution in the orbit plane of Jupiter.

/39

Let us first examine the total group of elliptical orbits with the same a and e , which are located uniformly around the sun in the given plane. Let us introduce two ellipses, whose axis forms the angle β . Let us turn one of the ellipses at the angle $d\beta$. Then its intersection with the other ellipse is shifted by the distance $d\ell$, with respect to this ellipse. Using r to designate the radius vector of the point of intersection and α to designate the angle between r and the tangent at this point, we shall have the obvious relationship

$$r \frac{d\beta}{2} = \sin \alpha d\ell.$$

Consequently, the quantity $d\ell$, which characterizes the frequency with which such orbits are intersected, is proportional to r , and inversely proportional to $\sin \alpha$. At the point of intersection, the angle between the orbits is apparently 2α . The shortest distance between contiguous orbits is proportional to

$$\frac{r}{\sin \alpha} \sin 2\alpha.$$

Therefore, for a uniform distribution of matter along the elliptical orbits, the surface density is proportional to

$$\frac{\sin \alpha}{r \sin 2\alpha}.$$

However, in stating that we are determining the density distribution in the orbit plane of Jupiter, we do not wish to say that it is necessary to determine the surface density. The density in space is determined from three measurements. Consequently, it is necessary to take the total group of planes located around the orbit plane of Jupiter, which correspond to such small values of i that it is possible to assume an equal distribution of surface density for all planes. All of these planes radiate out from the general point S. Due to this, in a certain volume which is located in the orbit of Jupiter, the density of *ceteris paribus* must be inversely proportional to its distance from the sun. Finally, we should note that the density cannot be the same in different parts of the elliptical orbit. In actuality, in order that our dust complex does not have periodic density fluctuations - as is assumed - it is necessary that the amount of matter be the same on each element of the ellipse which is penetrated by moving particles in a definite period of time. But, on the basis of the law of areas, we have

/40

$$\frac{r^2 d\beta}{2} = k \sqrt{p} dt,$$

where $p = a(1 - e^2)$ and k represent the gravitation constants.

Since

$$d\beta = \frac{2dt \sin \alpha}{r},$$

it then follows

$$r \sin \alpha dt = k \sqrt{p} dt.$$

Since the density along the same orbit changes as $\sim \frac{1}{dx}$ we consequently obtain a new factor of proportionality, namely:

$$\frac{r \sin \alpha}{\sqrt{p}}.$$

If we combine all of this together, we can see that in the total group of such orbits (with the same a and e) the density of cosmic matter changes proportionally to

$$\frac{\sin \alpha}{r \sin 2\alpha} \cdot \frac{1}{r} \cdot \frac{r \sin \alpha}{\sqrt{p}},$$

i.e.,

$$D = C \frac{\operatorname{tg} \alpha}{r \sqrt{p}},$$

where C is the proportional factor.

Let us now determine C in such a way that the total amount of matter distributed along the ellipse equals M - the mass of the decomposed comet, which is independent of the orbit elements.

/41

The amount of matter per unit orbit length is

$$\frac{C r \sin \alpha}{\sqrt{k^2 p}}.$$

Throughout the entire orbit, the total mass is, consequently,

$$\frac{C}{\sqrt{k^2 p}} \int r \sin \alpha dl = M,$$

or

$$\frac{C}{2\sqrt{k^2 p}} \int r^2 d\beta = M, \quad \text{or} \quad C \int dt = M,$$

and $CT = M$, if T is the rotation time along the given orbit.

Disregarding the factors which are constant for all the orbits, we can assume the following on the basis of the third law of Kepler:

$$C = \frac{M}{a^{3/2}}.$$

Thus, the density in the total group of similar orbits, located in close planes, is as follows, up to the coefficient of proportionality:

$$D = \frac{\operatorname{tg} \alpha}{r a^{3/2} \sqrt{p}}.$$

The angle α can be readily expressed by a , p and r . Investigating the triangle between a given point on the orbit on both focal points, we obtain

$$4a^2 e^2 = r^2 + (2a - r)^2 + 2r(2a - r) \cos 2\alpha,$$

from which it follows

$$\sin \alpha = \sqrt{\frac{ap}{2ar - r^2}} \quad \text{и} \quad \operatorname{tg} \alpha = \sqrt{\frac{ap}{2ar - r^2 - ap}}.$$

In order to determine the density in the entire collection of elliptical orbits which are characterized by all possible values of a and e , it is necessary to take the double integral D with respect to the variables a and e , attributing to each D the corresponding degree of probability. Consequently, we must first of all analyze the probability of encountering in the total ensemble the orbital elements a and e or a and p , which have a certain definite value. A simple solution of this problem is very difficult. I shall overcome this difficulty in the following way.

/42

Tisserand's criterion provides the dependence between comet elements before and after large perturbations.

We have

$$\frac{1}{a} + 2\sqrt{a(1-e^2)} \cos i = \frac{1}{a_1} + 2\sqrt{a_1(1-e_1^2)} \cos i_1.$$

In our case, periodic comets are obtained from non-periodic comets by Jupiter capturing the latter ones. Therefore, assuming

$$a_1 = \infty; \quad a_1(1 - e_1^2) = 2q; \quad a(1 - e^2) = p,$$

and also $i = i_1 = 0$, we obtain

$$\frac{1}{a} + 2\sqrt{p} = 2\sqrt{2q}.$$

We shall use ω to designate the angle which the tangent to the parabolic orbit of the comet makes with the orbit of Jupiter at the point where both orbits intersect. The tangents are drawn in the direction of orbital motion. Due to the properties of a parabola, it can be readily seen that $q = R \cos^2 \omega$, if R is the orbital radius of Jupiter. From this point on, we shall set $R = 1$.

The preceding formula yields:

$$\sqrt{p} = \sqrt{2} \cos \omega - \frac{1}{2a}.$$

In addition, Professor Newton (Ref. 33) derived the following relationship:

$$a = \frac{ps}{4m \cos \varphi \sin \gamma},$$

where $s = \frac{v_0}{v_1}$ is the ratio of the comet's velocities around the sun and Jupiter along its orbit, 2γ - the angle between the asymptotes of the comet's hyperbolic orbit with respect to Jupiter, φ - the angle between the perijovian of the relative orbit of the comet and the direction of motion of Jupiter, and m - the mass of Jupiter in units of solar mass. The sign of φ determines the type of transformed orbit.

/43

Expressing p , s , φ and γ by ω , d (the smallest distance between the orbits of Jupiter and the comet) and h (the distance at which Jupiter is located from the point of largest orbital encounter, when the comet is located at the corresponding point), Newton obtained the following relationship for the perturbed semi-major axis:

$$a = -\frac{s}{4m} \cdot \frac{A^2 + d^2 + h^2 \sin^2 \vartheta}{A \cos \vartheta \pm h \sin^2 \vartheta},$$

where

$$A = \frac{mR}{s^2}, \quad (R=1), \quad \cos \vartheta = \frac{1-s^2}{2s}; \quad s^2 = 3 - 2\sqrt{2} \cos \omega.$$

In this case, $d = 0$, since the comet intersects the orbit of Jupiter. After several transformations, we obtain:

$$4a = \frac{1 + \left(\frac{h}{m}\right)^2 2s^2 \sin^2 \omega}{\sqrt{2} \cos \omega - 1 + \left(\frac{h}{m}\right) 2s \sin^2 \omega}.$$

Thus, the variables a and p can be expressed by ω and h or q and h - quantities pertaining to the initial orbit. The problem regarding the probability of elements a and p is then replaced by

the similar problem with respect to h and q . What is the probability of h ?

Since Jupiter rotates along a circular orbit with a uniform velocity, for each moment of time - taken arbitrarily - the probability that it will be located at a definite point on the orbit does not depend on the position of this point. If select the time at which the comet intersects the orbit of Jupiter as this moment in time, we can state that the probability of the distance of Jupiter from the comet - i.e., h - does not depend on the magnitude of h . Only small values of h are of practical interest to us.

/44

The problem of the probability of q is somewhat more complex. It would appear that the simplest method is to solve the problem on the basis of direct observations, comparing the statistics for the magnitudes of q for all known periodic comets. A curve is also obtained which has minimum values of the ordinates close to the sun, reaching a maximum at a distance of the radius of the earth's orbit and then rapidly decreasing to zero. There is no doubt that it depends to a high degree on the visibility conditions of the comets at the moment of discovery, and this cannot be taken into account by a quantitative method. Therefore, by virtue of necessity we must resort to hypotheses and determine the probability of q on the basis of our assumptions (1).

Let us assume that at the initial moment on the boundary of the solar sphere of influence the comet coordinates will be:

$$x_0 = r_0; y_0 = 0; z_0 = 0,$$

and the velocity components v_0 , which form the angle δ with the radius vector r_0 , will be:

$$\left(\frac{dx}{dt}\right)_0 = -V_0 \cos \delta; \quad \left(\frac{dy}{dt}\right)_0 = V_0 \sin \delta; \quad \left(\frac{dz}{dt}\right)_0 = 0.$$

On the basis of the area integral we thus have:

$$r_0 V_0 \sin \delta = k \sqrt{2q},$$

and since $V^2 = \frac{2k^2}{r_0}$, we have

$$q = r_0 \sin^2 \delta.$$

Assuming that the comet is located in the plane of Jupiter's orbit, we have the following for the probability that the velocity vector will be included between δ and $\delta + d\delta$:

$$p_\delta = \frac{2d\delta}{\pi}.$$

But the probability for q to be located between q and $q + dq$ is p_δ , if only dq is determined from the relationship

$$dq = 2r_0 \sin \delta \cos \delta d\delta,$$

since all comets which fall in the interval δ and $\delta + d\delta$ must fall

/45

in the interval q and $q + dq$. It thus follows that the probability of p_q is:

$$p_q = \frac{dq}{\pi \sqrt{q(r_0 - q)}}.$$

The vector r_0 is a quantity which is immeasurably large as compared with q (perihelion distance of comets which can be captured by Jupiter). We can therefore assume that the desired probability of q is simply proportional to $\frac{1}{\sqrt{q}}$.

We shall now regard our partial density D as a function of two variables q and h , whose probability has been found. Consequently, the total density at a given distance from the sun is

$$\Delta = \iint D dh \frac{dq}{\sqrt{q}}.$$

Let us assemble our formulas for determining the density of cosmic dust in the orbital plane of Jupiter:

$$\begin{aligned} s^2 &= 3 - 2\sqrt{2} \cos \omega; \\ \sin \alpha &= \left(\sqrt{2} \cos \omega - \frac{1}{2a} \right) \sqrt{\frac{a}{(2a-r)r}}; \\ 4a &= \frac{1 + \left(\frac{h}{m} \right)^2 2s^2 \sin^2 \omega}{\sqrt{2} \cos \omega - 1 + \frac{h}{m} 2s \sin^2 \omega}; \\ D &= \frac{\text{tg } \alpha}{r \sqrt{pa^3 h}}; \quad \Delta = \iint D dh d \cos \omega. \end{aligned}$$

The double integral can be successfully calculated by mechanical quadrature, because α - which changes from 90° (perihelion) to a certain minimum, depending on e , and then again reaches 90° (aphelion) - exceeds by a factor of two the integrand at ∞ . This difficulty disappears if one accurate quadrature is used. This can be done simply with respect to h . If we assume

$$4a = \frac{1 + mx^2}{b + nx}$$

and if we express all other quantities as a function of x using this relationship - i.e., $\text{tg } \alpha$, $\sqrt{p}$ and $a^{3/2}$ - , then we obtain the radical from the polynomial of the seventh power in the denominator. It is impossible to accurately calculate such an integral.

In essence, this difficulty with the quadrature lies in the very nature of the problem, and cannot be eliminated by changing the variables in any way, if the preliminary density determination in the ensemble of elliptical orbits with identical a and e is retained.

There is one possible approach under these conditions: to calculate the density, not at a given point, but the mean density in a volume of finite magnitude.

For given a and p , the density is simply proportional to $\frac{\operatorname{tg} \alpha}{r}$. Let us determine the mean value for a circular surface $2\pi r_0 \Delta r$. This will be:

$$\frac{1}{2\pi r_0 \Delta r} \int \frac{\operatorname{tg} \alpha}{r} r dr d\vartheta = \frac{1}{r_0 \Delta r} \int \operatorname{tg} \alpha dr.$$

But

$$r - a = \pm \frac{\sqrt{a^2 \sin^2 \alpha - ap}}{\sin \alpha},$$

from which it follows that

$$dr = \frac{ap \cos \alpha d\alpha}{\sin^2 \alpha \sqrt{a^2 \sin^2 \alpha - ap}}.$$

Now assuming that $x = \cos \alpha$, for the integral we obtain the following relationship:

$$- \frac{p}{r_0 \Delta r} \int \frac{dx}{(1 - x^2) \sqrt{e^2 - x^2}},$$

where e is the eccentricity.

In addition, if we assume

$$\frac{x}{e} = \sin \varphi,$$

we can transform the integral to a new form

$$- \frac{p}{r_0 \Delta r} \int \frac{d\varphi}{1 - e^2 \sin^2 \varphi} = - \frac{p}{2r_0 \Delta r} \int \frac{d\varphi}{1 + e \sin \varphi} - \frac{p}{2r_0 \Delta r} \int \frac{d\varphi}{1 - e \sin \varphi}.$$

Finally, introducing the new variable

$$\operatorname{tg} \frac{\varphi}{2} = z,$$

we obtain a polynomial of the second power in the denominator of the integrand.

The first integral yields

$$- \frac{p}{r_0 \Delta r} \int \frac{dz}{1 + 2ez + z^2} = - \frac{p}{r_0 \Delta r} \frac{\operatorname{arctg} \frac{z+e}{\sqrt{1-e^2}}}{\sqrt{1-e^2}},$$

the second

$$- \frac{p}{r_0 \Delta r} \int \frac{dz}{1 - 2ez + z^2} = - \frac{p}{r_0 \Delta r} \frac{\operatorname{arctg} \frac{z-e}{\sqrt{1-e^2}}}{\sqrt{1-e^2}},$$

and on the whole we have:

$$\frac{1}{2\pi r_0 \Delta r} \int_{r_0 - \frac{1}{2} \Delta r}^{r_0 + \frac{1}{2} \Delta r} \frac{\operatorname{tg} \alpha}{r} r dr d\vartheta = - \frac{p}{r_0 \Delta r \sqrt{1-e^2}} \operatorname{arctg} \frac{2z \sqrt{1-e^2}}{1-z^2-e^2} \Big|_{r_0 - \frac{1}{2} \Delta r}^{r_0 + \frac{1}{2} \Delta r}$$

where

$$\cos \alpha = \frac{2ze}{1+z^2}; \quad \sin \alpha = \sqrt{\frac{ap}{(2a-r)r}}.$$

We should note that

$$\frac{dr}{d\alpha} = \frac{ap \cos \alpha}{(r-a) \sin^3 \alpha}.$$

If $\alpha = 90^\circ$, then $\frac{dr}{d\alpha} = 0$. Due to this, it is possible to represent the density at a very critical point in the form of a series consisting of powers of $\Delta\alpha$. Our mean density is:

/48

$$(D) = - \frac{1}{ar_0 \Delta r} \operatorname{arctg} \frac{2z \sqrt{1-e^2}}{1-z^2} \Big|_{r_0 - \frac{1}{2} \Delta r}^{r_0 + \frac{1}{2} \Delta r}$$

For any r_0 , we can calculate a , and then φ and z . In addition, for a given increment Δr , we can calculate Δz , according to the formula

$$\Delta z = - \frac{\sin^4 \alpha (r_0 - a) \Delta r}{2e \cos^3 \frac{\varphi}{2} \cos \varphi \cos \alpha \cdot ap}.$$

For $\alpha \neq 90^\circ$, the change in r is of one order of magnitude with α , and also z . Therefore, for a sufficiently small Δr , the integral

$$\frac{1}{r_0 \Delta r} \int_{r_0 - \frac{1}{2} \Delta r}^{r_0 + \frac{1}{2} \Delta r} \operatorname{tg} \alpha dr = \frac{\operatorname{tg} \alpha_m}{r_0 \Delta r} \int_{r_0 - \frac{1}{2} \Delta r}^{r_0 + \frac{1}{2} \Delta r} dr$$

differs slightly from $\operatorname{tg} \alpha_0$. Consequently, generally speaking, there is no necessity of employing the preceding expression with the exception of only critical points. For the latter, this inconvenient formula can be expanded in series.

We have:

$$\operatorname{arc tg} \frac{2z \sqrt{1-e^2}}{1-z^2} = 2z \sqrt{1-e^2} - \frac{2}{3} \sqrt{1-e^2} (1-4e^2) z^3 + \dots$$

But

$$\frac{2z}{1+z^2} = \frac{\cos \alpha}{e}.$$

Consequently,

$$z = \frac{\cos \alpha}{2e} + \frac{\cos^3 \alpha}{8e^3} + \dots,$$

and therefore

/49

$$\operatorname{arc tg} \frac{2z \sqrt{1-e^2}}{1-z^2} = \sqrt{1-e^2} \left[\frac{\cos \alpha}{e} + (1+2e^2) \frac{\cos^3 \alpha}{6e^3} + \dots \right].$$

But, by assumption, α differs very slightly from 90° . Introducing

$$\alpha = 90^\circ - \psi,$$

we obtain

$$(D) = - \frac{\sqrt{1-e^2}}{r_0 \Delta r a e} \psi \left(1 + \frac{1-e^2}{6e^2} \psi^2 + \dots \right) \Big|_{r_0 - \frac{1}{2} \Delta r}^{r_0 + \frac{1}{2} \Delta r}.$$

At the critical point

$$r = a(1+e) \quad \text{and} \quad r = a(1-e).$$

In the first case

$$(D) = \frac{\sqrt{1-e^2}}{r_0 \Delta r a e} \Delta \psi \left(1 + \frac{1+e^2}{6e^2} \Delta \psi^2 + \dots \right).$$

From the formula, we have

$$\sin \alpha = \sqrt{\frac{ap}{(2a-r)r}}$$

and, if r differs slightly from $a(1+e)$, we have:

$$\Delta \psi^2 = \frac{2e}{p} \Delta r - \left(\frac{1}{ap} + \frac{8e^2}{3p^3} \right) \Delta r^2 + \dots$$

Consequently:

$$\Delta \psi = \left(\frac{2e}{p} \Delta r \right)^{\frac{1}{2}} \left[1 - \left(\frac{1}{4ae} + \frac{2}{3} \frac{e}{p} \right) \Delta r + \dots \right].$$

The mean density is

$$(D) = \frac{\sqrt{1-e^2}}{r_0 \Delta r a e} \left(\frac{2e}{p} \Delta r \right)^{\frac{1}{2}} \left(1 + \frac{1+e^2}{6e^2} \cdot \frac{2}{p} \Delta r + \dots \right)$$

or, for sufficiently small Δr ,

$$(D) = \frac{\sqrt{1-e^2}}{r_0 a \sqrt{\Delta r} \sqrt{ae}} = \frac{\sqrt{2}}{ar_0 \sqrt{\Delta r} \sqrt{a(a-p)}}.$$

For $r = a(1 - e)$, we have the same formula.

It now remains to determine the maximum Δr , at which it is possible to remain at a sufficient distance from the critical points without changing the densities, calculated according to the simple formula $\frac{\text{tg } \alpha}{ra^{3/2}\sqrt{p}}$.

This problem is somewhat arbitrary, because the greater is Δr , the greater the distance from the critical points at which one must stop. Nevertheless, the mean density is obtained at those points which are most convenient for mechanical quadrature.

Let us represent the "instantaneous" density D in the form:

$$D = \frac{1}{ar \sqrt{2ar - r^2 - ap}},$$

and we shall try to determine its mean value

$$\begin{aligned} (D) &= \frac{1}{r_0 \Delta r a} \int_{r_0 - \frac{1}{2} \Delta r}^{r_0 + \frac{1}{2} \Delta r} \frac{dr}{\sqrt{2ar - r^2 - ap}} = \\ &= \frac{1}{r_0 \Delta r a} \left(\arccos \frac{a - r_0 - \frac{1}{2} \Delta r}{ae} - \arccos \frac{a - r_0 + \frac{1}{2} \Delta r}{ae} \right) \end{aligned}$$

Expanding in series in powers of Δr , we obtain:

$$\begin{aligned} D &= \frac{1}{r_0 \Delta r a} \left[\frac{\Delta r}{\sqrt{1 - \left(\frac{a - r_0}{ae}\right)^2} ae} + \right. \\ &\quad \left. + \frac{1}{24} \left(\frac{\Delta r}{ae}\right)^3 \frac{1 + 2 \left(\frac{a - r_0}{ae}\right)^2}{\left(1 - \frac{a - r_0}{ae}\right)^2 \sqrt{1 - \left(\frac{a - r_0}{ae}\right)^2}} \right]. \end{aligned}$$

Since

$$D_0 = \frac{1}{ar_0} \cdot \frac{1}{\sqrt{1 - \left(\frac{a - r_0}{ae}\right)^2}} \cdot \frac{1}{ae},$$

we then have

$$(D) = D_0 \left[1 + \frac{1}{24} \left(\frac{\Delta r}{ae}\right)^2 \frac{1 + 2 \left(\frac{a - r_0}{ae}\right)^2}{\left[1 - \left(\frac{a - r_0}{ae}\right)^2\right]^2} + \dots \right].$$

These series in powers of Δr are not suitable for determining the mean density at the critical points, but for all other cases they make it possible to determine the maximum Δr , at which (D) differs little from D_0 .

We should note that for $r = a(1 \pm e)$, it would be possible to use the accurate formula

$$(D) = \frac{1}{ar_0 \Delta r} \arccos \left(1 - \frac{1}{2} \frac{\Delta r}{ae} \right),$$

where

$$ae = \frac{1}{2} \sqrt{(2a)^2 - 4ap},$$

but the expression which we derived previously is more suitable for the calculations.

We should note that the result of integration has slight indeterminacy. Due to this fact, the calculated values of rD are insufficient for tracing the course of the integral curves close to the critical points. In particular, this is the case for the calculation of the first integral for $\frac{h}{m} = 1$ and $r = 1$, since we have here three critical points. The $\frac{h}{m}$ third is obtained for $\cos \omega = 0.7072$, as can be readily ascertained, if the expression $2a - 1 - ap$ is formulated on the basis of our formulas and if the following is substituted:

$$\cos \omega = \frac{1}{\sqrt{2}}.$$

Assigning different values to $\frac{h}{m}$ and $\cos \omega$, we can calculate the corresponding actual magnitudes of rD .

Determining individual values of rD for different $\frac{h}{m}$ and $\cos \omega$, we can calculate the critical points - determining them $\frac{h}{m}$ from the condition $\operatorname{tg} \alpha = \infty$ or $2a - r^2 - ap = 0$.

/57

I employed graphic interpolation for a definitive determination of the critical points. In addition, the first integrals were calculated in terms of the variable $\cos \omega$, and finally the second integrals were calculated in terms of the ensemble of first integrals. The results derived from the calculations are set forth above (see Table 6). The first integrals corresponding to $\frac{h}{m}$, which do not appear in the preceding Table, were determined by graphic interpolation.

Having a second integral for each r , we can obtain the desired Δ , dividing the magnitude of the integral by r . Assuming that the density of cosmic dust at the distance from the earth to the sun equals unity, we obtain the following Table 7.

TABLE 6

h/m	Critical Points $\cos \omega$	First Integrals	2nd Integral $r \int Ddh \cos \omega$	h/m	Critical Points $\cos \omega$	First Integrals	Second Integral
1	0.880	0.430	0.75	10	0.595	2.01	
2	0.860	0.140	1.0	30	0.420	1.35	
4	0.480	-0.040	1.5	100	0.390	0.833	
6	0.385	-0.100	2.0			0.470	
10	0.317	-0.150	2.5			0.298	
30	0.260	-0.200	3			0.180	
100	0.235	-0.215	4			0.145	
			6			0.0227	
			10			0.00340	
			20				
			100				
						$r = 0.25$	
						0.070	
						0.937	
						2.940	11.40
						0.75	
						1	
						2.460	
						1.734	
						1.430	
						1.237	
						0.780	
						0.384	
						0.260	
						0.200	
						0.151	
						0.0243	
						0.00414	
						$r = 0.35$	
						-0.5	
						0.112	
						0.376	
						0.0	
						0.5	
						0.625	
						0.75	
						0.825	
						1	
							11.12
							/52

TABLE 6 (continued)

h/m	Critical Points $\cos \omega$	First Integrals	Second Integral
1	—	1.562	
2	—	1.140	
4	—	0.895	
6	—	0.463	
10	—	0.270	
30	—	0.162	
100	—	0.0261	
		0.00418	
		$r = 0.50$	
1	—	0.178	
2	—	0.284	
4	—	0.493	
6	—	1.000	
10	0.850	2.321	
30	0.740	2.331	
100	0.715	1.401	
		1.120	10.88
		0.950	
		0.758	
		0.693	
		0.460	
		0.131	
		0.0276	
		0.00456	
		$r = 0.75$	
1	—	0.159	
2	—	0.225	
4	—	0.418	
6	—	0.740	
10	—	2.737	
30	0.903	2.460	
100	0.872	2.030	12.01

TABLE 6 (continued)

h/m	Critical Points $\cos \omega$	First Integrals	Second Integral
-0.5	0.955	$r = 1.25$ -0.5	7.83
0.0	0.94	0.0	
0.5	0.92	0.5	
1	-0.415	0.75	
2	-0.695	1.0	
4	-0.900	1.25	
6	-0.055	1.50	
10	-0.985	1.75	
30	-0.995	2	
100	-1.000	3	
		4	
		5	
		6	
		8	
		10	
		30	
		100	
-0.5	0.928	$r = 1.50$ -0.5	6.19
0.0	0.930	0.0	
0.5	0.850	0.5	
1	-0.458	0.75	
2	-0.500	1.00	
4	-0.835	1.25	
6	-0.922	1.75	
10	-0.985	2.0	
30	-0.993	2.5	
100	-1.000	4	
		6	
		8	
		10	
		30	
		100	

h/m	Critical Points $\cos \omega$	First Integrals	Second Integral
0.0	0.8505	$r = 2.00$ 0.0	5.16
0.5	0.7502	0.5	
1	0.550	1.0	
2	-0.015	1.5	
4	-0.700	1.75	
6	-0.855	2	
10	-0.945	2.5	
30	-1.000	3	
100	-1.000	4	
		5	
		6	
		8	
		10	
		30	
		100	

Close to the sun, the cosmic dust intensity decreases inversely proportional to the distance. At a distance of 3.90 astronomical units, there is a certain retardation, which is changed into a slight maximum around the orbit of Jupiter. In addition, the

/58

TABLE 7

r in radii of Jupiter's orbit	r in radii of earth's orbit	Second Integrals	Density Δ
0.05	0.26	10.84	3.75
0.10	0.52	11.41	1.974
0.15	0.78	10.74	1.229
0.25	1.30	11.40	0.789
0.35	1.82	11.12	0.550
0.50	2.60	10.88	0.376
0.75	3.90	12.01	0.277
1.00	5.20	16.66	0.288
1.25	6.50	7.83	0.108
1.50	7.80	6.19	0.071
2.00	10.40	5.16	0.045

density rapidly decreases, and asymptotically approaches zero (Figure 3)

There is no doubt that in such a complex of cosmic dust the observed brightness will rapidly increase as the sun is approached, just as occurs for zodiacal light. However, it is quite doubtful whether such a mechanism for the capture of matter can provide sufficient density of it in space, since the probability for large perturbations is, generally speaking, very small. In addition, the elliptical orbits which are produced exist for only a limited period of time. The density of the dust-like matter, obtained in the prescribed way, must rapidly decrease along both sides of the orbital plane of Jupiter, which is not the case in actuality. In order to clarify this problem, it is necessary to study the manner in which the probability for comet capture is related to the inclination of their orbits to the orbital plane of Jupiter. If it is found that this probability decreases very rapidly with inclination, then a search must be made for other possible methods by which the planetary system can be enriched with cosmic matter. In any case, the transformation of non-periodic comets into periodic comets, and the decomposition of the latter into meteoric streams, represent an indisputable fact which has been repeatedly

/59

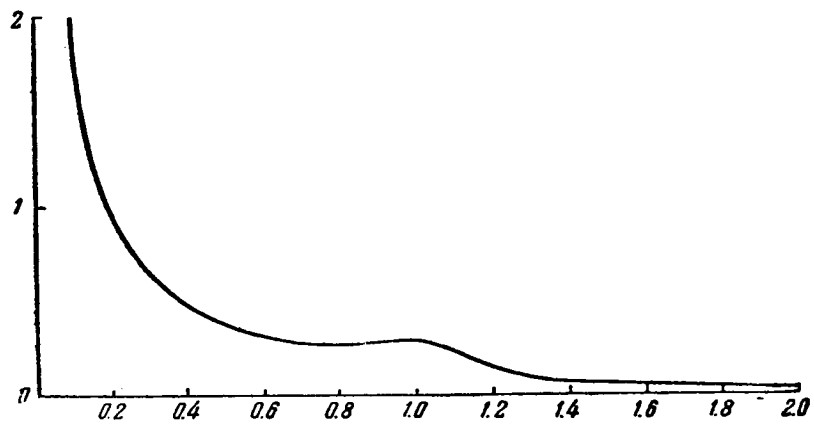


Figure 3

Density of Cosmic Dust in a Complex of Comets Captured by Jupiter.

observed in actuality.

CHAPTER 4

The Probability for the Capture of Comets as a Function of Their Orbital Inclination

In order to solve the formulated problem, let us examine the following model. Let a disturbed planet - Jupiter - describe a circular orbit around the sun, which is located at the origin. Let ρ be the radius for the sphere of action of Jupiter. If a parabolic comet passes through the sphere of action of Jupiter, it undergoes large perturbations and can be captured. The sphere of action describes a certain torus during the motion of Jupiter. Jupiter can be located at any point on its orbit with equal probability. It thus follows that the probability for large perturbations must be proportional to the time during which the comet is located within the torus or - which is the same thing - the section of its orbit included within the torus. Thus, in order to solve the formulated problem it is necessary to determine the mean magnitude of this section for all possible orientations of the comet's orbit in its plane, and all of its possible perihelion distances.

/60

If the orbit of Jupiter, having radius a , is located in the plane xy then the torus equation will be

$$(r - a)^2 + z^2 = \rho^2; \quad r^2 = x^2 + y^2.$$

The equation of the parabola will be

$$\left(x \sin \omega - y \frac{\cos \omega}{\cos i}\right)^2 = 4q \left(q - x \cos \omega - y \frac{\sin \omega}{\cos i}\right),$$

where i is the orbit inclination, q and ω - the perihelion distance and angle between the line of apsides and the x -axis. Finally, the equation for the orbital plane of the comet is $z = y \tan i$.

/61

When solved together these three equations give the coordinates $x_1, y_1, z_1; x_2, y_2, z_2$ for the point at which the torus intersects the parabola.

A rigorous solution is very complex. In order to simplify the problem, we shall assume that - due to the smallness of ρ as compared with a - the section of the parabola within the torus differs insignificantly from a straight line.

In addition, it can be assumed that the curves of intersection of the torus with the orbital plane are ellipses if the angle of the orbital inclination is not too small.

The equation of this ellipse in the orbital plane of the comet, which is assumed to be the xy plane, is

$$(x-a)^2 + y^2 \sin^2 i = \rho^2$$

or, assuming

$$\begin{aligned} x &= a + \xi; y = \eta, \\ \frac{\xi^2}{\rho^2} + \frac{\eta^2}{\rho^2/\sin^2 i} &= 1. \end{aligned} \quad (8)$$

Let us set $a = 1$.

The equation for the parabola can be written in the form

$$(x \sin \omega - y \cos \omega)^2 = 4q(q - x \cos \omega - y \sin \omega).$$

Disregarding terms of a higher order with respect to ξ and η , we find the following equation for a straight line which represents the section of the parabola within the torus:

$$A\xi + B\eta + c = 0, \quad (9)$$

where

$$\begin{aligned} A &= 2 \sin^2 \omega + 4q \cos \omega; & B &= -2 \sin \omega \cos \omega + 4q \sin \omega; \\ C &= \sin^2 \omega + 4q \cos \omega - 4q^2. \end{aligned}$$

Solving equations (8) and (9) with respect to ξ and η , we can determine the points at which the parabola intersects the ellipse, and we can then find the length of the corresponding section d:

/62

$$\begin{aligned} \frac{d^2}{4} [(2q \sin \omega - \sin \omega \cos \omega)^2 + (\sin^2 \omega + 2q \cos \omega)^2 \sin^2 i]^2 = \\ = (4q^2 + \sin^2 \omega)(2q \sin \omega - \sin \omega \cos \omega)^2 \rho^2 + \\ + (\sin^2 \omega + 2q \cos \omega)^2 \rho^2 \sin^2 i - (\sin^2 \omega - 4q^2 + 4q \cos \omega)^2 \sin^2 i. \end{aligned}$$

The magnitude of d, determined from this expression, represents the function $f(i, \omega, q)$. In order to determine the corresponding probability of p_i , which is dependent only upon i , it is necessary to integrate $f(i, \omega, q)$ with respect to all possible values of q and ω . The probability of q is proportional to $q^{-1/2}$. The probability of ω does not depend on its magnitude. Consequently, we have

$$p_i \sim \iint f(i, \omega, q) \frac{d\omega dq}{\sqrt{q}}.$$

It is necessary to find the integration limits for q and ω . We can first write

$$d^2 = 4 \rho^2 \frac{(1 - M \cos^2 i) - N \sin^2 i}{(1 - M \cos^2 i)^2},$$

where

$$M = \frac{(\sin^2 \omega + 2q \cos \omega)^2}{4q^2 + \sin^2 \omega}; \quad N = \frac{(\sin^2 \omega - 4q^2 + 4q \cos \omega)^2}{4q^2 + \sin^2 \omega}$$

We must now find the total group of values of q and ω , for which $d^2 > 0$. These quantities can be regarded as coordinates in the orbital plane of the comet, which define a certain area. Consequently, it can be stated that the condition $d^2 \geq 0$ is satisfied each time that the perihelion of the comet orbit falls within the limits of the corresponding area. It can be readily seen that the relationship holds

$$\sin^2 \omega - 4q^2 + 4q \cos \omega = 0 \quad \text{or} \quad \cos \omega = 2q - 1$$

if the comet trajectory passes through the point with the coordinates $x = 1, y = 0$. The equation for the same curve passing through the point $1 + \xi$ is

$$\cos \omega + \eta \sin \omega = 2q - 2q\xi - 1,$$

where higher order terms are discarded. This is an equation with two parameters ξ, η , which are related to each other by the relationship

/63

$$\xi^2 + \eta^2 \sin^2 i = \rho^2.$$

By varying the variables ξ and η , we can determine the series of curves, the enveloping of which determines the region of actual values of d . Excluding η between these two equations, we find

$$f(q, \omega, \xi) = \cos \omega - 2q + 1 + \sin \omega \frac{\sqrt{\rho^2 - \xi^2}}{\sin i} + 2q\xi = 0.$$

Finally, excluding ξ between $f(q, \omega, \xi)$ and

$$\frac{\partial f}{\partial \xi} = 0 = \frac{\sin \omega}{\sin i} \frac{\xi}{\sqrt{\rho^2 - \xi^2}} - 2q,$$

we find the desired equation for the enveloping regions of actual values of q, ω - namely:

$$F(\omega, q) = \cos \omega - 2q + 1 + \rho \frac{\sqrt{\sin^2 \omega + 4q^2 \sin^2 i}}{\sin i} = 0.$$

On the basis of this equation, after certain transformations we can determine the following approximate expression:

$$2q = 1 + \cos \omega \pm \rho \sqrt{\frac{\sin^2 \omega}{\sin^2 i} + (1 + \cos \omega)^2} + \dots$$

Assigning fixed values of i and substituting different values of q , we can find ω , which represent the desired integration limits. The integral itself must be calculated by mechanical quadratures. According to Tisserand (Ref. 34), for the radius of Jupiter's sphere of action we can assume $\rho = 0.062$, and we can calculate the values

of the integrand for q as 0.0, 0.25, 0.50, 0.75, 1.00, and for ω 0°, 15°, 30°, 45°, 60°, 90°. However, this method cannot be applied for $i = 0$. In this case, the intersection of the torus with the plane of the comet orbit represents the combination of two concentric circles with the radii $1 + \rho$ and $1 - \rho$. Due to the smallness of ρ , the section of the parabola within the torus can be regarded in this case as the segment of a straight line, since the smallest radius of curvature for the parabola - close to its perihelion - is approximately ten times greater than the orbital radius of Jupiter. If q differs significantly from $a = 1$ - namely, if $q \leq 0.8$ - we can find the following expression for the length of the parabola segment within an accuracy of small quantities of the fifth order:

/64

$$d = 2\rho \sqrt{\frac{1}{1-q}} \left[1 + \frac{1}{8} \frac{q\rho^2}{(1-q)^2} + \dots \right]$$

If, on the other hand, $q = 1$, we then have the exact expression

$$d = \sqrt{\rho^2 + 4\rho}.$$

The method given here is only approximate. However, it can be shown that its accuracy is not less than several percents for the assumed value for the radius of the sphere of action.

If we designate the probability for a comet being captured by Jupiter by p_0 for a normal angle of inclination, we obtain the following Table which represents the final computational results:

Angle of inclination i	Probability of capture p_0
90°	1.00
60	1.20
45	1.45
30	2.02
15	3.98
0	62.7

It can be seen that an overwhelming amount of matter which is produced by the disintegration of comets captured by Jupiter must be located close to the plane of its orbit. A complex of cosmic dust which is produced by the same mechanism in the solar system cannot have anything in common with the observed form of zodiacal light - which, as is known, is significantly extended on both sides of the ecliptic plane. Therefore, it is necessary to investigate another possible mechanism by which cosmic dust occupies the solar system. In this respect, the ring of asteroids is of particular interest.

/65

These bodies - which frequently exhibit comet-like orbits - have arbitrarily small dimensions. Due to the negligible force of gravity, all disintegration processes - for example, asteroids colliding with each other and with meteors - produce cosmic dust which enters interplanetary space. Due to this, the problem of the total number and over-all surface of asteroids must be investigated in greater detail.

CHAPTER 5

Possible Number and Total Surface of Asteroids

Generally speaking, observations have shown that the number of asteroids increases with a decrease in their brightness. The lower limit of brightness for asteroids which have been discovered is determined by the optical power of the astrographs employed. Asteroids which are brighter than 12.5 mg are apparently becoming increasingly exhausted, and are discovered only very rarely. There is far from adequate information about weaker bodies, and the smaller the number, the weaker they are - as can be seen from the following Table.

/66

NUMBER OF SMALL PLANETS WITH KNOWN ORBITS AS A FUNCTION OF BRIGHTNESS

mg	8.5	8.5-9.5	9.5-10.5	10.5-11.5	11.5-12.5
No.	5	13	53	154	219
%	0.4	0.9	3.8	11.0	15.6
mg	12.5-13.5		13.5-14.5	14.5-15.5	> 15.5
No.	457		357	119	23
%	32.6		25.5	8.5	1.6

There is no doubt that the decrease in the number of weak planets is due to the difficulty entailed in locating them and in performing repeated observations to determine the orbits. 1513 asteroids were known on January 1, 1941, whose orbits were determined. On January 1, 1943, 1546 asteroids were known (Ref. 35). The orbits have been determined for only 10-15% of the planets which have been discovered; the remaining planets have been lost. This occurrence is clearly indicated by the photographs of small planets which were made by Hubble and Baade with a 100" reflector at the Mount Wilson Observatory. Baade found traces of 37 asteroids with a 100" reflector on 21 films - i.e., 4.4 asteroids, on the average, per square degree. Consequently, in the $\pm 5^\circ$ zone the total number of asteroids which can be detected by the 100" reflector - i.e., up to approximately the 19th magnitude - must be 44,000. According to Hubble, on the basis of his photographs of the sky using the 100" reflector, it can be concluded that the total number of asteroids up to 19 mg amounts to 30,000 throughout the entire sky (Ref. 36).

/67

Let N_m designate the total number of asteroids up to m apparent magnitude in the mean opposition. Plotting $\lg N_m$ as a

function of m on a graph, we can see that the resulting curve passes considerably lower than the number given by Hubble and Baade. This enables us to correct the course of the curve $\lg N_m$ beginning with $m = 12.5$ and continuing it up to $m = 19$.

As a result, we obtain the following numbers:

	$\lg N_m$		$\lg N_m$
m to 8.5	0.70	m to 14.5	3.80
9.5	1.26	15.5	4.10
10.5	1.85	16.5	4.35
11.5	2.40	17.5	4.50
12.5	2.90	18.5	4.60
13.5	3.40		

This amounts to about 40,000 asteroids up to $m = 18.5$, which corresponds approximately to the figure given above.

The application of the theory of perturbations to secular changes in the orbital elements represents an entirely different source of information regarding the number of asteroids. As is known, the secular acceleration of the perihelion for Mars cannot be explained only by the interaction between different planets, similarly to the secular acceleration of the perihelion for Mercury. Due to the great distance ^{/68} of the former planet from the sun, any effects arising from the theory of relativity are out of the question. According to Lever'ye, the total mass of asteroids must amount to 0.1 of the terrestrial mass, in order to explain the inequality in the perihelion of Mars. Gartsler found 1/6 of the terrestrial mass; Ostin found approximately the same figure.

If we accept the estimate given by Lever'ye and Gartsler for the total mass of asteroids, it can be shown that the overwhelming mass of matter in space between the orbits of Mars and Jupiter must occur in a state of finely-crushed fragments.

For this purpose, let us estimate the mass of an asteroid M , which corresponds to its apparent brightness in the mean opposition m .

We shall regard the asteroid as a spherical body having the radius ρ at a distance from the sun r and from the earth Δ .

Let its albedo be A , and let the reflection of light follow the law of Lambert.

Let j be the apparent brightness of the asteroid for the

given phase α ; $j_{\odot}$ - the brightness of the sun seen from the earth at the distance R .

It can be readily shown (Ref. 37) that

$$\frac{j}{j_{\odot}} = \frac{2}{3} \frac{A}{\pi} \frac{R^3 \rho^3}{\Delta^3 r^2} [\sin \alpha + (\pi - \alpha) \cos \alpha].$$

In opposition $\alpha = 0$, and therefore

$$\frac{j}{j_{\odot}} = \frac{2}{3} A \frac{R^3 \rho^3}{\Delta^3 r^2}.$$

For the mean opposition, we set $r = 3R$, and therefore $\Delta = 2R$. Consequently, we have

$$\frac{j}{j_{\odot}} = \frac{2}{3} A \frac{\rho^3}{36 R^4} = \frac{1}{54} A \frac{\rho^3}{R^4}.$$

If δ is the asteroid density, then its mass is

$$M = \frac{4}{3} \pi \delta \rho^3 \approx 4 \delta \rho^3.$$

Excluding ρ , we have

$$\left(\frac{j}{j_{\odot}}\right)^{1/3} \frac{54^{1/3}}{A^{1/3}} R^3 = \frac{M}{4 \delta}.$$

Replacing the brightness by stellar magnitudes, we find:

$$M = 10^{1/5} (m_{\odot} - m) \frac{54^{1/5}}{A^{1/5}} 4 \delta R^3.$$

It is relevant to set

$$A = 0.3; \delta = 3; R = 150\,000\,000 \text{ km}.$$

We obtain

$$M = 10^{44} \cdot 10^{1/5} (m_{\odot} - m) \text{ grams}$$

This formula provides, for example, the following logarithms of the asteroid mass for different values of the apparent stellar magnitude m (assuming $m_{\odot} = -26.5$):

m	$\lg M$	m	$\lg M$
3.5	26	33.5	8
8.5	23	38.5	5
13.5	20	43.5	2
18.5	17	48.5	-1
23.5	14		
28.5	11		

If the mean stellar magnitude of asteroids pertains to the unit of distance from the sun and from the earth, then we must set $r = \Delta = R$, and we have

$$M = 10^{1/4} \cdot (m_{\odot} - m) \frac{R^{3/4}}{(2/4)^{1/4}} \pi^{1/2} \text{ grams}$$

i.e.,

$$M = 4.5 \cdot 10^{41} \cdot 10^{1/4} \cdot (m_{\odot} - m) \text{ grams}$$

On the basis of this formula, we can estimate the total mass of asteroids, whose number we can estimate with a correction in conformity with the estimates of Hubble and Baade. For each unit interval of stellar magnitude $\Delta m = 1$, we obtain (Table 8)

/70

TABLE 8

m	No. of asteroids ΔN	Asteroid Mass M	Total Mass $\Delta N \cdot M$
9	13	$5.0 \cdot 10^{22}$	$6.5 \cdot 10^{22}$
10	53	$1.3 \cdot 10^{22}$	$6.9 \cdot 10^{22}$
11	179	$3.2 \cdot 10^{21}$	$5.7 \cdot 10^{22}$
12	544	$7.9 \cdot 10^{20}$	$4.35 \cdot 10^{22}$
13	1716	$2.0 \cdot 10^{20}$	$3.4 \cdot 10^{22}$
14	3800	$5.0 \cdot 10^{19}$	$1.9 \cdot 10^{22}$
15	6300	$1.3 \cdot 10^{19}$	$8.2 \cdot 10^{21}$
16	9800	$3.2 \cdot 10^{18}$	$3.2 \cdot 10^{22}$
17	9200	$7.9 \cdot 10^{17}$	$7.2 \cdot 10^{21}$
18	8200	$2.0 \cdot 10^{17}$	$1.6 \cdot 10^{21}$
Total Mass			$3 \cdot 10^{24} \text{ gr.}$

On the other hand, the total mass of an asteroid ring, defined as 0.1 of the terrestrial mass, amounts to $6 \cdot 10^{26}$ grams.

An asteroid having an apparent magnitude of 19 mg has a radius of about 3 km in diameter, and represents the observational limit utilizing the most powerful, present-day devices at this distance. The Table given above shows that the total mass of individual asteroids up to the 19th magnitude is only about 1% of the total mass of the asteroidal ring. The remaining 99% must occur in a more pulverized state.

More detailed conclusions cannot be substantiated. It is apparent, however, that if we formulate the problem of determining the limiting stellar magnitude, corresponding to the total mass M of the entire asteroid ring, on the basis of equation

$$\int_{m_1}^{m_2} M \frac{dN}{dm} dm = M,$$

then - even attributing a maximum value to MdN - we obtain im-
probably large values for m , with which the equation loses any
meaning.

/71

On the basis of the preceding statements, it can be asserted
that not only is the overwhelming portion of the asteroid ring in
the form of fine fragments, but also these finely-crushed fragments
are distributed according to an entirely different law than is the
case for large asteroids. The reason for this can be found in the
fact that these particles are crushed additionally as the result
of collisions. The finely-crushed particles which are formed
during such collisions are thrown out into space, in all proba-
bility, at a small velocity with respect to the asteroid, and
gradually occupy space. The probability for the collisions must
be approximately proportional to the total surface of the corres-
ponding asteroidal bodies.

Small asteroids are of significant importance in this respect.
Let us estimate the total surface of asteroids having different
categories of brightness.

On the basis of the preceding formulas, we have:

$$\rho^2 = 180 R^2 10^{3/4} (m_{\odot} - m),$$

where

$$m_{\odot} = -26.5; R^2 = 2.2 \cdot 10^{26} \text{ cm}^2.$$

For different m , this yields:

m	ρ^2	$\rho^2 N$	m	ρ^2	$\rho^2 N$
9	$6.3 \cdot 10^{13}$	$82 \cdot 10^{12}$	15	$2.5 \cdot 10^{11}$	$157 \cdot 10^{13}$
10	$2.5 \cdot 10^{13}$	$132 \cdot 10^{12}$	16	$1.0 \cdot 10^{11}$	$98 \cdot 10^{13}$
11	$1.0 \cdot 10^{13}$	$179 \cdot 10^{13}$	17	$4.0 \cdot 10^{10}$	$37 \cdot 10^{13}$
12	$4.0 \cdot 10^{12}$	$218 \cdot 10^{13}$	18	$1.6 \cdot 10^{10}$	$13 \cdot 10^{13}$
13	$1.6 \cdot 10^{12}$	$275 \cdot 10^{13}$	19	$6.3 \cdot 10^9$	
14	$6.3 \cdot 10^{11}$	$259 \cdot 10^{13}$			

This total surface is of the same order of magnitude as the
lunar surface, since for the latter we have

/72

$$\rho^2 = 16 \cdot 10^{16} \text{ cm}^2.$$

For smaller asteroids, the total surface must be much
larger.

Let us estimate the total surface and mass of small planets
within the limits of the 19-48th stellar magnitude, and let us
assume that the same total mass - equalling $2 \cdot 10^{21}$ grams -

corresponds to each interval of stellar magnitude.

We have:

$$\begin{array}{lll}
 m = 9-18; & \Sigma M \Delta N = 3 \cdot 10^{24} \text{ r; } & \Sigma \rho^2 \Delta N = 1.4 \cdot 10^{16} \text{ cm}^3 \\
 19-28 & 2 \cdot 10^{22} & 4.2 \cdot 10^{16} \\
 29-38 & 2 \cdot 10^{22} & 4.2 \cdot 10^{16} \\
 39-48 & 2 \cdot 10^{22} & 4.2 \cdot 10^{16}
 \end{array}$$

It was shown in the statements above that approximately the same total mass of asteroidal matter corresponds to each interval of stellar magnitude. In this case, the total surface of these asteroids must increase per each 10 stellar magnitudes by approximately a factor of one hundred. Because the overwhelming mass of the asteroids must lie in the region of weak magnitudes, it is clear that their total surface must be enormous. Thus, the numerous asteroids which are less than a kilometer, or even a meter in size, are primarily the ones which disintegrate during subsequent collisions with meteors.

CHAPTER 6

Effect of Asteroid Disintegration

Various considerations lead to the conclusion that a dust-like /73
substance is continuously accumulating in the solar system.

In particular, a study of the photometric and radiometric properties of the lunar surface shows that the planets which have no atmosphere are surrounded by a thick layer of dust. It has been known for a long period of time that any element on the lunar surface reaches the greatest apparent brightness only during the full moon, when a reflected light ray coincides with the line of vision and when, consequently, there are no shadows. For other positions, which even correspond to the normal incidence of light, the apparent brightness of a surface element is much less. This fact can be interpreted by assuming that the lunar surface abounds in irregularities, the majority of which cannot be seen even by the most powerful telescopes, but which, nevertheless, reflect the shadows and lower the surface brightness. The floor of the lunar sea - which seems to be completely flat in a telescope - must be spirated with cracks or can be pitted by extremely small craters. On the other hand, observations on the temperature change on the lunar surface during lunar eclipses definitely show that the thermal conductivity coefficient for the lunar soil is negligible (Ref. 38), at least 1000 times smaller than the thermal conductivity of grass, sand, or granite. Not one terrestrial substance, in which /74
heat is propagated by molecular transfer, can be even remotely compared with the material comprising the lunar surface. In order to explain this incongruity, it must be assumed that matter on the lunar surface occurs in an extremely disintegrated state, and that heat is transferred by direct radiation. Only a pulverized substance, located in a vacuum, can have such low thermal conductivity as the lunar surface.

Since the photometric properties of the lunar surface do not differ significantly from the properties of Mercury's surface, it can be assumed that all the planets which have no atmosphere are surrounded by a layer of finely-crushed substance, are extremely irregular, and abound in a great number of depressions.

This cosmic dust which covers the surface of planets was not formed, apparently, when they first existed. It must have gradually arisen as they evolved.

It is natural to think that the same cause underlies the origin of

cosmic dust on the surface of planets which have no atmosphere, in the narrow zone of the asteroidal ring, and throughout interplanetary space surrounding the sun. This cause can lie in cosmic meteors, which are passing through the solar system, continuously impacting on the surface of the planets. These meteors, which have a very high kinetic energy, gradually break up the hard rocks which comprise the surface layers of the planets having no protective atmospheric layer. If a planet is sufficiently large, the dust which is thus produced remains on its surface, and gradually accumulates from century to century. On the other hand, if the force of gravity on the surface is negligible, the rising dust enters interplanetary space at a large or small relative velocity, with respect to the planet.

The intensity of this process, by which interplanetary space accumulates dust, depends on the probability of the planet encountering meteors, and primarily occurs in the region of the asteroidal /75 ring because - according to the statements given above - the total surface of small asteroids is extremely large.

Depending on the magnitude of the relative velocity, the ejected particles are propagated to a greater or lesser extent in all directions from the asteroidal zone, and even occupy all of interplanetary space under certain conditions. If this process takes place very slowly, the role of radiative deceleration must be taken into account.

From this point on, we shall base our discussion on purely dynamic considerations, assuming that each particle moves around the sun exclusively under the influence of its attraction.

Let us postulate the following model for the formation of a complex of cosmic dust comprising the zodiacal light. Let a planet move around the sun on a circular orbit, and let dust particles be ejected from this planet continuously and uniformly in all directions with the definite velocity v_0 . This process continues for a period of time which considerably exceeds the duration of this planet's rotation around the sun. It is necessary to find the relative density of the ejected matter at any point in interplanetary space. It is apparent that the complex of matter which is produced will have rotational symmetry with respect to the normal of the orbital plane which passes through the sun. Due to this, the same result is obtained if the density of matter at a definite point in space is found either as the sum of the densities produced by each orbital element of the generatrix planet, or as the sum of the densities taken at each circumference element perpendicular to the axis of symmetry of the model and which are produced by only one orbital

element of the generatrix planet. Let us adopt the latter model, fix the position of the planet on its orbit, and pass through it the x-axis with the origin at the sun S. The coordinates of the planet O will be R, 0, 0, the same coordinates of the point P at which the density x, y, z is determined.

/76

According to this, we must determine the density at any point on a circle passing through P, with the axis coinciding with the general axis of symmetry for the system SZ.

The elliptical trajectory passing through P is determined by the initial conditions, for which we take the initial coordinates and velocity with respect to magnitude and direction of the ejected particle. Since the initial coordinates, which coincide with the position of the generatrix of the planet, are fixed, for each value of absolute ejection velocity v_1 we have only two parameters c_1, c_2 , which determine the described trajectory. Let the ejection take place at the moment $t = 0$. During the time t , the particle reaches $P(x, y, z)$, so that its coordinates will be single-valued functions of the parameters c_1, c_2 and the time t . We thus have the functions $x(t, c_1 c_2); y(t, c_1 c_2); z(t, c_1 c_2)$. For certain other values of the initial parameters, we shall have

$$\begin{aligned} x(t, c_1 + \Delta c_1, c_2), \\ x(t, c_1, c_2 + \Delta c_2), \\ x(t, c_1 + \Delta c_1, c_2 + \Delta c_2). \end{aligned}$$

Thus, we obtain four points in space, which differ very little with respect to position and which are reached by the particles simultaneously at the moment t , but for somewhat different values of the parameters.

These points form a parallelogram within small quantities of the second order. The trajectories of contiguous particles, which have almost the same velocity v , pass through the apex of this parallelogram. Per unit of time, the particles describe a parallelepiped, whose base represents the area of the given parallelogram, and the length of the generatrices - the velocity v at a given point. The volume of the parallelepiped characterizes the degree to which matter is pulverized in space, and the greater the density of matter, the smaller this volume.

Let a particle be ejected for definite values of the parameters c_1, c_2 and pass through a given point in space x, y, z . Let the probability for these parameter values be

/77

$$P_1, P_2.$$

The resulting density of matter in space is proportional to

$$\rho = \sum \sum \frac{P_1 P_2}{a^{3/2} \text{объем}(x, x_1, x_2, x_{12}, \tau)} \dots \quad (1)$$

x, x_1, x_2, x_{12} designate the coordinates for the parallelogram apexes. The factor $a^{3/2}$ is introduced in the denominator because the orbits are independent of the mass of matter.

Let matter be continuously ejected from the producing planet for a rather long period of time T .

Particles, which have once passed through the volume in space under consideration, will again pass through it after expiration of the time τ - the period of rotation along the elliptical orbit - and in the interval T such passages will be

$$n = \frac{T}{\tau},$$

which is, generally speaking, an arbitrarily large number. Thus, all of the hyperbolic orbits, which can provide only one single passage at most, can be completely disregarded. The mean density, which is determined for the entire interval under consideration, will be proportional to n or $\frac{T}{\tau}$.

According to the third law of Kepler,

$$\frac{4\pi^2 a^3}{\tau^2} = GM$$

and, consequently,

$$\tau = \frac{2\pi a^{3/2}}{\sqrt{GM}},$$

which determines the factor $a^{3/2}$ in the denominator of the density expression. /78

Let us examine those velocities with which particles are ejected from the producing planet, which enable them to approach the sun. In order to do this, the ejection velocity must be greater than the orbital velocity of the producing planet.

The origin is located at the sun S ; the x -axis represents the radius vector of the planet for the moment of ejection; the y -axis is directed parallel to the tangent in the direction of motion for the planet; the z -axis - perpendicular to the orbital plane.

The geometric location of the vector points of relative velocity is a sphere, described by the radius r_0 of the planet from its center. The geometric position of the vector points for the absolute particle ejection velocities is the same sphere

with the initial point located at the distance u from the center, in an opposite direction to the motion of the planet 0. Let us use ϕ , ψ to designate the angles which determine the directions of the relative velocity vectors with respect to the planet, while ψ to the longitude and ϕ corresponds to the latitude. For the absolute velocity components, we have:

$$v_x' = v_0 \cos \varphi \cos \psi; v_y' = v_0 \cos \varphi \sin \psi + u; v_z' = v_0 \sin \varphi.$$

The orbital plane of the ejected particle passes through the points O, S, P and intersects the sphere which represents the geometric position of the points v_0 , along the circumference of a small circle at the distance $u \sin i$ from the center.

Since the orbital plane passes through the radius vector R (points O and S), the inclination of the orbit i is determined by the simple expression

$$\operatorname{tg} i = \frac{z}{y}.$$

In the orbital plane of the ejected particle, the geometric position of the points for the absolute velocity vectors is, apparently, a small circle having the radius

$$v = \sqrt{v_0^2 - u^2 \sin^2 i} = \sqrt{v_0^2 - \frac{u^2 z^2}{y^2 + z^2}}.$$

/79

The initial point of the vectors is located in the plane of the circle at the distance $u \cos i$ from the center. By determining the inclination of the particle orbit, we can find the magnitude of the initial absolute velocity, join the vector at a certain angle χ up to its intersection with the given circle. We shall stipulate that the angle χ is measured from the extended radius vector.

The application of our general formula (1) for determining the density of matter at any point in space and for determining the system of spatial isophotes is a very complex problem, which entails great amount of work in performing the numerical calculations. We shall confine ourselves to calculating the distribution of density in the orbital plane of the producing planet, and also in the direction of the axis of symmetry in a perpendicular direction.

Not wishing to restrict the discussion, we can assume that the radius of the planet orbit, the mass of the sun, the gravitation constant, and consequently the circular velocity of the planet

equal unity

$$R=1, M=1, G=1, u=1.$$

The direction of the relative velocity vector is determined by the angles $\psi, 0$. Let v_1 be the corresponding absolute velocity. We have

$$v_1 = \sin \psi + \sqrt{v_0^2 - \sin^2 \psi},$$

and in addition, for the semimajor axis and the parameter we obtain

$$\frac{1}{a} = 2 - v_1^2; \quad p = v_1^2 \sin^2 \psi.$$

Let us examine the point in the orbital plane with the coordinates r, θ . The true anomaly θ_0 corresponding to this point is given by the expression

$$r = \frac{p}{1 + e \cos(\theta_0 - \theta)}.$$

In addition, we have

$$e \cos \theta_0 = p - 1; \quad e^2 = 1 - \frac{p}{a}.$$

Let us now examine the contiguous point of ejection

/80

$$(\psi + d\psi, 0).$$

The particle will describe an orbit which is adjacent to the preceding one. The ejection velocity $v_1 + dv_1$ differs from the previous one by the quantity dv_1 , where

$$dv_1 = \left(1 + \frac{\sin \psi}{\sqrt{v_0^2 - \cos^2 \psi}} \right) \cos \psi d\psi.$$

We can determine the new values

$$a + da, \quad p + dp, \quad \theta_0 + d\theta.$$

The corresponding orbit intersects the extended radius vector r at the contiguous point at the distance δr .

It is apparent that

$$r + \delta r = \frac{p + dp}{1 + (e + de) \cos(\theta_0 + d\theta_0 - \theta)},$$

and the angle θ remains positive. Within an accuracy of small quantities of the second order, the new trajectory will be parallel

to the previous one and passes at the distance

$$2r \sin \beta,$$

from it, where β is the angle between the tangent and the radius vector,

$$\sin \beta = (ap)^{-\frac{1}{2}} (2ar - p^2)^{-\frac{1}{2}}$$

Let us now examine the ejection of a particle at the angle ψ , $d\varphi$, so that the particle slightly leaves the orbital plane of the planet. It is apparent that within an accuracy of small quantities of the second order, it can be assumed that the absolute ejection velocity is the same as the previous one. Only the inclination of the particle orbit is changed by the quantity

$$di = \frac{d\varphi}{\sin \psi},$$

but in this orbit the particle describes absolutely the same ellipse. Thus, in order to determine the new position of the particle, it is only necessary that the ellipse, which was determined previously, turn at the angle di around the radius R . The point, corresponding to the previous r , θ , has thus increased by

/81

$$dz = r \sin \theta di = r \sin \theta \frac{d\varphi}{\sin \psi}.$$

We thus obtain three infinitely close points in space, through which almost parallel trajectories pass. The corresponding volume, described per unit of time, is

$$r \sin \theta \frac{d\varphi}{\sin \psi} \sin \beta \delta r v,$$

which corresponds to the ejection of particles at an infinitely small solid angle $d\varphi d\psi$.

Let P_ψ be the probability of ejection per unit of solid angle. The density of matter in space will be

$$P_\psi d\varphi d\psi \sin \psi \left(r \sin \theta \sin \beta v \frac{dr}{d\varphi} a^{1/2} d\varphi d\psi \right)^{-1}$$

If it appears that for two or more values of ψ , the particles pass through the same volume of space, we shall take the sum of the given expressions. In order to obtain a definitive value for the density at a point located at the distance r from the sun, it is necessary to average over all the values of the angles ϑ , regarding partial densities as equally probable. The total density will thus be

$$D = \int \left(\sum \frac{P_{\psi} \sin \psi}{r \frac{dr}{d\psi} \sin \vartheta \sin \beta v a^{1/4}} \right) d\vartheta,$$

where ϑ changes from 0 to 2π .

In practice, it is more advantageous to perform calculations for the argument ψ , and not ϑ . Let us assign several values to ψ . Assigning one definite value to the relative velocity v_0 , we can calculate several values for the corresponding auxiliary quantities: /82

ψ_1	ψ_2	ψ_3	ψ_4	p_1	p_2	p_3	p_4
v_1	v_2	v_3	v_4	ϑ_{01}	ϑ_{02}	ϑ_{03}	ϑ_{04}
a_1	a_2	a_3	a_4	ϑ_1	ϑ_2	ϑ_3	ϑ_4

We can thus establish the numerical relationship between ψ and ϑ and, ultimately, we can use ϑ to express all the quantities depending on ψ .

We must now average the density over all values of the relative velocity v_0 . Let the probability of v_0 be P_{v_0} . In a special case, which will be considered at a later point, we can assume that

$$P_{v_0} = A e^{-k^2(v_0 - v_m)^2}; v_m \approx 0,$$

assuming that the relative velocities are grouped primarily around a certain mean value v_m , which can be rather small. Thus, we finally have

$$\bar{D} = \int D P_{v_0} dv_0.$$

The problem has thus been primarily solved.

The method which has been presented, however, is too complex for numerical calculations.

Another method can be proposed, which is simpler in the numerical respect and which leads to the same results with respect to the density distribution in the plane of the generatrix planet. The idea underlying this method is as follows: Instead of infinitely close trajectories corresponding to different ejection conditions for a given position of the planet, we shall first examine infinitely close trajectories with the same elliptical elements, but which are produced from very different elements in the planetary orbit.

We shall thus begin our examination with trajectory orbits of particles which are produced under identical conditions from all points on the planetary orbit.

Let us examine a group of elliptical trajectories in the orbital plane with identical values of a and p . Let us take a point A at a distance r from the sun, lying on one of these trajectories. Let us turn the axis of this ellipse at an infinitely small angle $\beta d\chi$ in the orbital plane. The point A shifts to a new position B, through which a contiguous trajectory passes. Attention should be paid to the fact that all vectors of r pass through S and, consequently, represent a bundle which separates into space. Due to this, the factor r^{-1} enters into the density expression. The shortest distance between contiguous orbits is evidently,

/83

$$\overline{AB} \cos \beta = r \cos \beta d\chi,$$

where β is the angle of the tangent and the radius vector.

The volume V , which is described per unit of time by particles which move along contiguous trajectories, changes with distance in proportion to the expression

$$r \cos \beta \cdot r \cdot v.$$

According to the second law of Kepler,

$$rv \sin \beta = \sqrt{p}$$

and therefore

$$V = r \operatorname{ctg} \beta \sqrt{p}.$$

As was indicated above, the condition must be introduced that the total mass of matter in orbit does not depend on its elements. Therefore, for partial density we obtain the simple expression

$$D = \operatorname{tg} \beta r^{-1} p^{-\frac{1}{2}} a^{-\frac{3}{2}}.$$

Such a density would occur in a complex of particles moving exclusively along ellipses with the elements a and p . For a given relative velocity, this corresponds to one definite value of the ejection angle ψ . If the ejection probability for this angle is p_ψ , then the resulting density is given by the integral

/84

$$\bar{D} = \int \operatorname{tg} \beta p_\psi a^{-\frac{3}{2}} p^{-\frac{1}{2}} r^{-1} d\psi.$$

In order to obtain the total density corresponding to all possible velocity values, it is necessary to take just one integral.

It is not difficult to calculate the ejection probability as the function of the angle γ of the absolute velocity vector. For relative velocity, all ejection directions are, by definition, equally probable. Let P_γ be the ejection probability with a certain absolute velocity v_1 at the angle γ . It is apparent that the terminals of all the vectors entering the zone $\cos \gamma d\gamma$ also fall into the zone $\cos \psi d\psi$, which is investigated from the center of the absolute velocity vectors. Consequently, we have

$$P_\psi \cos \psi d\psi = P_\gamma \cos \gamma d\gamma,$$

from which it follows

$$P_\gamma = P_\psi \frac{\cos \psi d\psi}{\cos \gamma d\gamma}.$$

However,

$$v_0 \cos \psi = v_1 \cos \gamma.$$

Therefore, assuming that

$$\frac{u}{v_0} = k, \quad \frac{v_1}{v_0} = k \sin \gamma + \sqrt{1 - k^2 \cos^2 \gamma},$$

we find

$$P_\gamma = \frac{v_1^2 \sin \gamma}{v_0^2 \sin \psi + v_0^2 \frac{u}{v_1} \cos \psi \cos \gamma}$$

or after various transformations

$$P_\gamma = \frac{(k \sin \gamma + \sqrt{1 - k^2 \cos^2 \gamma})^2 \sin \gamma}{\left\{ 1 - \cos^2 \gamma \left[k^2 \sin^2 \gamma + 1 - k^2 \cos^2 \gamma + 2 k \sin \gamma (1 - k^2 \cos^2 \gamma)^{\frac{1}{2}} \right] \right\}^{\frac{1}{2}} + k \cos^2 \gamma}.$$

This expression is very complex. If $k = 1$ in the special case, i.e.,

$$v_0 = u,$$

we then have

/85

$$P_\gamma = \frac{4 \sin^2 \gamma \cos \gamma}{1 - \sqrt{1 - 4 \sin^2 \gamma \cos^2 \gamma} + \cos^2 \gamma} = 4 \sin \gamma.$$

If k differs slightly from unity, so that $k = 1 + \epsilon$, with $\epsilon \ll 1$, then - retaining in the expansion only the first power of ϵ - for

angles γ which are not too close to 0° , we find that

$$P_\gamma = 4 \sin \gamma \left[1 + \left(1 - \operatorname{ctg}^2 \gamma - \frac{\operatorname{ctg}^4 \gamma}{\cos^3 \gamma} \right) \varepsilon \right].$$

On the other hand, if $k = 0$, we then have

$$P_\gamma = \text{const.}$$

If k is small, then - as can be readily seen

$$P_\gamma = 1 + 2k \sin \gamma.$$

Finally, taking the fact into account that

$$\cos 2\beta = \frac{2ar - r^2 - 2ap}{(2a - r)r}; \quad \operatorname{tg}^2 \beta = \frac{ap}{2ar - r^2 - ap},$$

we find the definitive expression for density in the form

$$D = \int \frac{P_\gamma d\gamma}{ar \sqrt{2ar - r^2 - ap}}.$$

In order to calculate this integral, it is most advantageous to assign different values of γ , calculating the auxiliary values by the formulas given above, which can be combined for purposes of convenience:

$$v_1 = \sin \gamma + \sqrt{v_0^2 - \cos^2 \gamma}; \quad \frac{1}{a} = 2 - v_1^2; \quad \sqrt{p} = v_1 \sin \gamma.$$

It is interesting to note that the cosmic dust density approaches infinity at the orbital distance of the generatrix planet for any values of the relative velocity $v_0 \geq \sqrt{3}$. The case of $v_0 = 1$ represents an exception to this, when the density at this distance remains finite, although it reaches a maximum value. In actuality, if $v_0 > u$, the probability P_γ for the ejection of particles at the angle γ does not become zero at any point. Consequently, the problem of the finiteness of the integral expressing the density value can be reduced to studying the behavior of the pole /86

$$(2r - r^2 a^{-1} - p)^{-\frac{1}{2}}.$$

Let us investigate the function

$$f(\gamma) = 2r - r^2 a^{-1} - p.$$

Let γ_0 be the special value of γ , for which $f(\gamma_0) = 0$. If $\gamma - \gamma_0 = \varepsilon$ is small, then the function $f(\gamma)$ can be expanded in series in powers of ε

$$f(\gamma) = f(\gamma_0) + \varepsilon f'(\gamma_0) + \frac{\varepsilon^2}{2} f''(\gamma_0) + \dots$$

In order that the integral is finite, it is necessary that the first derivative does not become zero simultaneously with the function itself, i.e., that $f'(\gamma_0) \neq 0$.

If, at any rate, the first derivative becomes zero, then the integral for the corresponding value of γ_0 becomes infinite.

We have

$$2r(1-r) + (r^2 - \sin^2 \gamma_0)(\sin \gamma_0 + \sqrt{v_0^2 - \cos^2 \gamma_0})^2 = 0$$

and

$$f'(\gamma_0) = \cos \gamma (\sin \gamma + \sqrt{v_0^2 - \cos^2 \gamma}) F(\gamma, v_0),$$

while $F(\gamma_1, v_0) \neq 0$.

Generally speaking, these equations are incompatible, i.e., the density integral has a finite value. However, for $r = 1$ the first equation yields

$$\cos \gamma_0 (\sin \gamma_0 + \sqrt{v_0^2 - \cos^2 \gamma_0})^2 = 0,$$

from which it follows that

$$\gamma_0 = 270^\circ,$$

since for $v_0^2 \geq 3$ the 90° angle is excluded as it does not conform to an elliptical orbit. In addition, for the same value of γ_0 the second equation becomes zero. Consequently, for $r = 1$ and $v_0^2 \geq 3$ the integral has the form

$$\int \frac{dz}{\varepsilon};$$

and for $\varepsilon \rightarrow 0$ it approaches infinity, similarly to the logarithm. The case $v_0 = u$ must be analyzed separately. The ejection probability will thus be

$$P_\gamma = \sin \gamma,$$

/87

and therefore the integral is transformed into the form

$$\overline{Dr} = 2 \int \frac{(2 \cos^2 \gamma - 1)^{1/2} d \cos \gamma}{\sqrt{r - r^2(2 \cos^2 \gamma - 1) - 2(\cos^2 \gamma - 1)^2}},$$

For the special case of $r = 1$, we find

$$\overline{Dr} = \sqrt{2} \int_0^{\pi/4} (2 \cos^2 \gamma - 1)^{1/2} \cos \gamma^{-1} d\gamma.$$

This integral is finite, and can be readily calculated. It will be shown at a later point that the value of this integral, for $r = 1$, represents a maximum as compared with contiguous distances from the sun.

It has thus been demonstrated that no matter how the asteroids are broken down or no matter what the ejection velocities, a significant portion of the cosmic dust must be concentrated in the region of the asteroids. Thus, this region in the solar system, along with individual small planets, must abound in pulverized particles of matter.

This provides an explanation of the zodiacal band which occupies the entire sky and which can be seen quite well under favorable conditions, especially as this band, which is visible in spite of the great distance of the reflecting particles, is spatially different from the other portions of the zodiacal light due to the great number of these reflecting particles.

In actuality, it is quite probable that the so-called counter glow is not genetically related to the entire zodiacal band, and that the latter is equally not related to the wide cones of zodiacal light.

Let us now calculate the density distribution of cosmic dust in the orbital plane of the generatrix planet.

/88

We shall start from the basic integral

$$\overline{Dr} = 2 \int \frac{P_\gamma d\gamma}{\sqrt{2r - r \cdot a^{-1} - \rho}} \cdot \frac{1}{a^{1/2}}.$$

First of all, for each definite value of relative velocity through equidistant values of the ejection angles, we shall calculate the auxiliary functions appearing under the integral sign. For example, for $v_0^2 = 2$ we have

γ	270	260	250	240	230	220	210	200	190	180	170	160
$\frac{d\psi}{d\gamma}$	0.29	0.30	0.32	0.35	0.39	0.46	0.55	0.68	0.84	0.90	1.17	1.32
P_γ	0.070	0.083	0.097	0.112	0.136	0.177	0.240	0.346	0.500	0.707	0.984	1.296
a^{-1}	1.829	1.824	1.813	1.791	1.756	1.703	1.618	1.489	1.293	1.000	0.390	0.040
ρ	0.171	0.170	0.165	0.156	0.143	0.122	0.095	0.060	0.031	0.000	0.042	0.229

It is rather difficult to calculate the integral close to the pole, where the integrand becomes infinite. The results derived from calculating D for different v_0^2 are as follows:

TABLE 9

r	$v_0^2=1$	$v_0^2=2$	$v_0^2=3$	$v_0^2=4$
0.2	2.50	2.55	1.20	0.63
0.4	1.00	1.00	1.00	1.00
0.8	0.43	0.46	0.31	1.54
1.0	0.57	∞	∞	∞
1.2	0.32	—	0.25	1.24
1.6	0.09	—	0.052	0.41

What is the density of the medium at a very small distance from the sun?

Let us examine the general case when

/89

$$v_0 > u.$$

The probability of P_γ is everywhere finite, and $a^{-3/2}$ does not become zero at any point. For $r \rightarrow 0$, we have

$$r^2 a^{-1} \ll 2r$$

$$Dr = \int P_\gamma a^{-1/2} (2r)^{-\frac{1}{2}} \left(1 - \frac{p}{4r}\right)^{-1} d\gamma,$$

while

$$p = v_1^2 \sin^2 \gamma; \quad v_1 = \sin \gamma + \sqrt{v_0^2 - \cos^2 \gamma}$$

are always finite quantities. Therefore, the angle γ approaches 0° . Let $\gamma = \varepsilon$, where ε is a small quantity. Consequently, we have

$$Dr^{1/2} \sqrt{2} = (3 - v_0^2)^{1/2} P_\gamma(\varepsilon_0) \varepsilon_0.$$

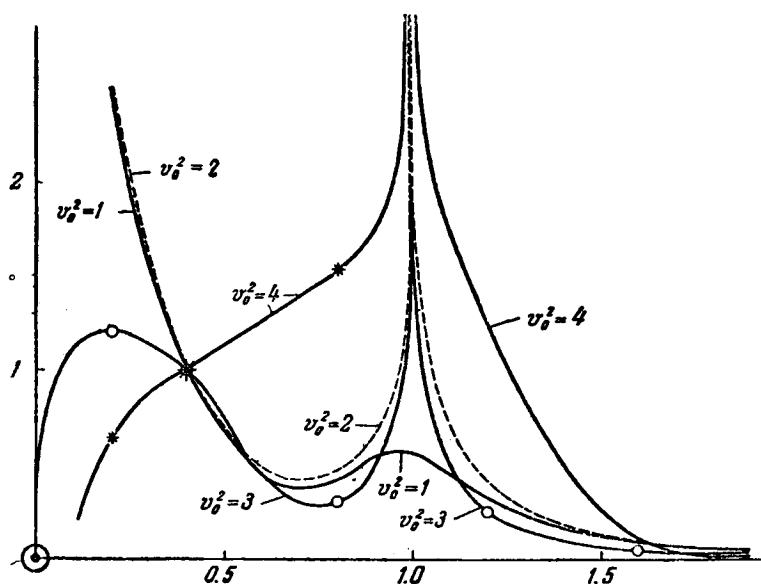


Figure 4

Cosmic Dust Density in a Complex of Matter Ejected by an Asteroid.

The important assumption can thus be drawn that all values of γ from 0° to 90° are possible. /90

We have $P \propto \epsilon^2$. In addition, when ϵ changes from 0 to ϵ_0 , r changes from 0 to $r_0 = \frac{P}{2}$. Therefore, $D \propto \frac{1}{r}$. Thus, at a very small distance from the sun, the density of matter changes in a manner which is inversely proportional to the first power of distance.

If we take very large values of r and examine the density in a complex of elliptical orbits which are close to parabolic orbits, the corresponding values of the angles γ will be close to 45° . The first term r remains under the radical in the expression, and therefore the density of matter, which is otherwise very small due to the presence of $(2 \cos^2 \gamma - 1)^{3/2}$ in the numerator, will change as $r^{-3/2}$.

Finally, if we examine a complex of hyperbolic orbits and if we assume that the ejection velocity is very large, then the probabilities of P_γ can be fairly accurately regarded as constant, independent of the ejection angle. In addition, it is apparent that

a and p are also constant for a given relative velocity. Therefore, up to the proportionality factor we have

$$\overline{Dr} = \frac{1}{a \sqrt{2ar - r^2}} \int_0^{\gamma_0} \frac{d\gamma}{\sqrt{1 - k^2 \sin^2 \gamma}},$$

where

$$k^2 = \frac{v_1^2}{2r - 2r^2 + r^2 v_1^2},$$

and γ_0 determines the region in which possible γ change

$$k \sin \gamma_0 = 1.$$

If $k \leq 1$, we then have

$$\overline{Dr} = \frac{1}{a \sqrt{2ar - r^2}} \int_0^{\frac{\pi}{2}} \frac{d\gamma}{\sqrt{1 - k^2 \sin^2 \gamma}}.$$

If $k \geq 1$, then - introducing the new variable

/91

$$k \sin \gamma = \sin \gamma_1,$$

we find

$$\overline{Dr} = \frac{1}{a \sqrt{2ar - r^2}} \frac{1}{k} \int_0^{\frac{\pi}{2}} \frac{d\gamma_1}{\sqrt{1 - \frac{1}{k^2} \sin^2 \gamma_1}}.$$

If v_1 is very large, then $k = \frac{1}{r}$. Therefore,

$$\overline{Dr} \approx \int_0^{\frac{\pi}{2}} \frac{d\gamma}{\sqrt{1 - r^2 \sin^2 \gamma}}.$$

The latter integral asymptotically approaches $\frac{\pi}{2} \cdot \frac{1}{r}$ for an unlimited increase in r . Therefore, at a large distance from the sun, the density of matter decreases inversely proportional to the square of the distance.

These are the general results which are attained in a general investigation of the density distribution of ejected matter in the orbital plane of a generatrix planet. We can see that at a small

velocity the ejected matter naturally remains in the zone of the asteroids, and does not reach the sun. For ejection velocities which are comparable to the orbital velocity of the planet, the density close to the sun decreases approximately in inverse proportion to distance. The density decrease then slows down, reaches a minimum, again increases, and approaches infinity at the distance of the planetary orbit. The decrease occurs more and more rapidly, and finally the density asymptotically approaches zero. For relative ejection velocities of about $\sqrt{3}$ and for large velocities in the vicinity of the sun, a rarefaction is produced, which constantly increases, and finally the entire phenomenon is reduced to the narrow asteroidal zone.

It is interesting to note that in the region of distances $0.2 R - 0.8 R$ the density distribution for all values of v_0^2 in the interval 1 - 2 is almost exactly the same. Assuming a probability for the ejection velocity distribution which is proportional to

/92

$$e^{-k \cdot v_0^2}, k \gg 1,$$

we find the resulting density, calculated for all possible velocities, which is close to the corresponding density observed in the zodiacal light.

Let us now examine the case in which a particle is ejected perpendicularly to the orbital plane, and let us set ourselves the problem of determining the density distribution of cosmic matter along the z-axis. It can be readily seen that in the given case the absolute ejection velocity does not depend on the angle α with the radius vector, and is given by the simple expression

$$v_1^2 = v_0^2 - u^2 = v_0^2 - 1.$$

Since the probabilities for all v_0 are the same, the vectors v_1 in the vertical plane are also equal and equally probable.

Taking the fact into account that

$$R=1; \quad M=1; \quad G=1; \quad u=1,$$

we obtain

$$v_1^2 \sin \alpha = p.$$

In particular, for $\alpha = 90^\circ$ we have

$$v_1^2 = p = z.$$

Let us find p for other angles α .

We have

$$R[1 + e \cos(\vartheta + \vartheta_0)] = p,$$

where ϑ , ϑ_0 represent the amplitude and direction of the major axis. In addition,

$$z[1 + e \sin(\vartheta + \vartheta_0)] = p.$$

It thus follows that

$$\left(\frac{p}{R} - 1\right)^2 + \left(\frac{p}{z} - 1\right)^2 = 1 - \frac{p}{a}, \quad R=1,$$

or

/93

$$p^2 \left(1 + \frac{1}{z^2}\right) - p \left(v_1^2 + \frac{2}{z}\right) + 1 = 0.$$

Thus, we find the following relationship between the distance z on the vertical axis, at the point at which the trajectory is intersected, and the ejection angle α

$$\sin^2 \alpha = \left(0.5 + \frac{1}{v_1^2 z}\right) \frac{1}{1 + \frac{1}{z^2}} + \frac{\sqrt{0.25 + \frac{1}{v_1^2 z} - \frac{1}{v_1^4}}}{1 + \frac{1}{z^2}}, \dots (2)$$

while the plus sign must be retained in front of the radical. Varying α , we obtain different values of z , and vice-versa.

The preceding expression determines the angle α in all quadrants. Forward and backward motion does not play a role in determining the nature of the orbit. The angle, taken in the second or fourth quadrant, determines the trajectory inclination, with intersection of the z -axis, which is symmetrical with respect to the first and third quadrant. This does not influence the results. Consequently, we can confine ourselves to investigating the angles α in only one quadrant—for example, the first quadrant—reading the angle from 0° to 90° from the extended radius vector in the direction of the extended z -axis. Let

$$\frac{z}{v_1^2} = \eta.$$

We have

$$\sin^2 \alpha = (1 + 0.5\eta v_1^4) \frac{\eta}{1 + v_1^4 \eta^2} + \frac{v_1^2 \eta^{3/2}}{1 + v_1^4 \eta^2} \sqrt{1 - \eta + 0.25 v_1^4 \eta}.$$

For small values of v_1^2 , we obtain

$$\sin^2 \alpha = \eta (1 + v_1^2 \sqrt{\eta - \eta^2}).$$

Thus, for different η and v_1^2 , by an accurate formula we find the following angles α (Table 10).

TABLE 10

/94

$\eta \backslash v_1^2$	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0.00	0°	18.4°	26.6°	33.2°	39.2°	45.0°	50.8°	56.8°	63.4°	71.6°	90.0°
0.01	0.0	18.5	26.6	33.3	39.3	45.1	50.9	57.0	63.7	71.8	90.0
0.05	0.0	18.6	26.9	33.6	39.8	45.7	51.6	57.8	64.6	72.9	90.0
0.10	0.0	18.7	27.1	34.0	40.4	46.5	52.4	58.7	65.7	74.0	90.0
0.20	0.0	19.0	27.7	34.9	41.5	47.8	54.2	60.7	67.8	75.7	90.0
0.50	0.0	—	29.5	—	45.0	—	58.8	—	72.8	—	90.0
1.00	0.0	21.5	32.6	42.1	50.5	58.3	65.4	71.9	78.3	83.2	90.0

Let us now calculate the density of the medium at different distances from the sun, along the z-axis according to our general theory. The initial conditions determining the particle ejection are the velocity v_1 and the direction angles α and γ . The angle α represents the angular distance of the absolute velocity vector above the orbital plane of the planet. The angle γ corresponds to the azimuth, and is calculated in the orbital plane in the positive direction of the y-axis and the x-axis.

In the case which we are considering, the direction of the vector v_1 is determined by the angles α , 0, while the ejected particle intersects the z-axis at the distance z at the angle β - as was shown above.

This angle is given by the expression

$$zv_1^2 \sin \beta = \sqrt{p},$$

where

$$v_1^2 = \frac{2}{z} - \frac{1}{a}.$$

Therefore, we have

$$\sin^2 \beta = \frac{p}{z^2 \left(v_1^2 + \frac{2}{z} - 2 \right)},$$

where

$$p = v_1^2 \sin \alpha.$$

In the same vertical plane, let us now change the ejection angle by the quantity $d\alpha$. The corresponding trajectory intersects the z-axis at a point, at a distance dz from the preceding one.

/95

The derivative $\frac{dz}{d\alpha}$ can be readily determined from the equation given above.

In the case of a small absolute velocity, we have

$$v_1^2 \sin^2 \alpha = z$$

and therefore

$$\frac{dz}{d\alpha} = 2 \sin \alpha \cos \alpha v_1^2.$$

Let us now orient the absolute velocity vector in the direction $(\alpha, \gamma + d\gamma)$, - i.e., retaining the previous inclination to the orbital plane - which is inclined from the vertical plane by the angle $d\gamma$. The particle thus describes an orbit which is inclined toward the vertical one at the infinitely small angle

$$d\chi = d\gamma \operatorname{ctg} \alpha.$$

It is necessary to determine the coordinates of the point at which the corresponding particle trajectory intersects the plane YOZ. The problem can be solved using the previous relationships, if they are only applied to the trajectory plane of the particle. Thus, the ejection angle α can be assumed to be the same as before ($\alpha = \alpha_1$), but the velocity has changed somewhat. Actually, we find

$$v_0^2 = u^2 + v_1^2 - 2u v_1 \cos \alpha d\gamma.$$

Consequently,

$$v_1 = u \cos \alpha d\gamma + \sqrt{(u \cos \alpha d\gamma)^2 + v_0^2 - u^2}$$

or

$$v_1^2 = v_0^2 - u^2 + \Delta,$$

where

$$\Delta = 2u \sqrt{v_0^2 - u^2} \cos \alpha d\gamma.$$

In the vertical plane, we have

$$v_1^2 = v_0^2 - u^2.$$

Thus, Δ represents an increase in v_1^2 for a deviation from the vertical plane. Using this new value of v_1 , we can find the distance from the origin in the plane of the inclined particle orbit. Then, taking the magnitude of the inclination into account, we can find the desired rectangular coordinates.

/96

As a result, we obtain the following coordinates for the intersection of three contiguous trajectories with the plane YOZ:

y	z
0	z
0	$z + \frac{z}{\alpha} d\alpha$
$z \operatorname{ctg} \alpha d\gamma$	$z + K d\gamma$

The coefficient K, which is of no importance below, is

$$K = 2 \frac{z^2 \left[v_1^2 \sin^2 \alpha \left(1 + \frac{1}{z^2} \right) + v_1^2 - \frac{1}{z} \right]}{v_1 \left(\frac{v_1^2 \sin^2 \alpha}{z} - 1 \right)} \cos \alpha.$$

The doubled volume, which is described per unit of time in the spatial region close to the point (0, 0, z) by all particles ejected in the interval of the angles $d\alpha$, $d\gamma$ - i.e., within the limits of the area of a sphere

$$d\alpha d\gamma \cos \alpha,$$

will be

$$z \operatorname{ctg} \alpha d\gamma \frac{dz}{d\alpha} d\alpha v_1 \sin \beta$$

or

$$v_1 \cos \alpha \frac{dz}{d\alpha} d\alpha d\gamma.$$

Consequently, the volume corresponding to the unit of a solid angle will be

/97

$$v_1 \frac{dz}{d\alpha}.$$

For all α and ϕ , the ejection probabilities around the vertical

plane are identical. Therefore, the density distribution of matter for a given v_1 along the z -axis will be

$$D = \frac{1}{v_1} \frac{d\alpha}{dz} \frac{1}{a^{3/2}}.$$

Since the resulting density is obtained for all possible velocities, each of which corresponds to the ejection probability P_{v_1} , we have

$$D = \int \frac{1}{v_1} \frac{d\alpha}{dz} P_{v_1} dv_1 \frac{1}{a^{3/2}}.$$

This expression can be readily calculated by mechanical quadrature, if we assign the form of the function P_{v_1} .

Let us calculate this for the case of a small absolute velocity, when it can be assumed - as was indicated above - that

$$D = \frac{1}{v_1^3 \sin \alpha \cos \alpha a^{3/2}} = \frac{1}{v_1 \sqrt{z(v_1^2 - z)}} \cdot \frac{1}{a^{3/2}}.$$

In addition, let us assume that

$$P_{v_0} = A e^{-k^2(v_0 - v_m)^2}; \quad v_m \approx 0.$$

The resulting density is

$$D = \int P_{v_0} \frac{dv_1}{dv_0} dv_0 \frac{1}{v_1} \frac{d\alpha}{dz},$$

while

$$v_1 dv_1 = v_0 dv_0.$$

Thus, we have

$$D = \int \frac{v_1}{v_1^2} \frac{dz}{dz} P_{v_0} \frac{1}{a^{3/2}} dv_0,$$

where

$$a^{-3/2} = (2 - v_1^2)^{3/2}, \quad v_1^2 = v_0^2 - 1.$$

This expression is general in nature.

In the case of small velocities for v_1 and $v_m \approx 0$, we have

$$D = A \int_{\sqrt{u^2+z}}^{\sqrt{3}} \frac{v_0 dv_0 (3-v_0^2)^{1/2} e^{-k^2 v_0^2}}{(v_0^2-1) \sqrt{z(v_0^2-1-z)}};$$

or, changing the variables, we have

$$D = \int_z^3 \frac{d\omega}{\omega} e^{-k^2 \omega} \frac{(z-\omega)^{1/2}}{\sqrt{z(\omega-z)}}, \quad z \neq 0.$$

For practical calculations, we can take

$$D = \int_z^2 + \int_z^{z+\epsilon} + \int_{z+\epsilon}^2,$$

while $\epsilon \ll z$ is a small quantity. Consequently,

$$D = \frac{e^{-k^2 z} (2-z)^{1/2}}{z^{1/2}} \left[2\sqrt{\epsilon} - \left(\frac{1}{2-z} + \frac{2}{3} k^2 + \frac{2}{3} \frac{1}{z} \right) \epsilon^{1/2} \right] + \\ + \int_{z+\epsilon}^2 \frac{d\omega}{\omega} \frac{(2-\omega)^{1/2}}{\sqrt{\omega-z}} \frac{e^{-k^2 \omega}}{\sqrt{z}}.$$

The value of ϵ is usually assumed to be 0.01 - 0.02. The first term is of the same order of magnitude as the second. For example, let us set $k^2 = 1$ and $k^2 = 10$.

As the result of the calculations, we have the following values for the first and second terms in arbitrary units for different values of z (Table 11).

TABLE 11

/99

	$k^2 = 1.0$		$k^2 = 10.0$	
$z = 0.04$	12.8	21.0	8.13	7.95
0.08	5.79	13.56	2.59	4.47
0.16	3.52	6.87	0.78	0.66
0.32	1.63	3.29	0.0826	0.0423
0.64	0.259	1.07	0.000791	0.00262

Let us conditionally assume that the density equals unity for $z = 0.16$. For the same cases, we have the following definitive values of the resulting density (Table 12).

TABLE 12

	$k^2 = 1.0$	$k^2 = 10.0$
$z = 0.04$	6.49	22.3
0.08	2.61	6.9
0.16	1.00	1.00
0.32	0.33	0.061
0.64	0.064	0.00118

Using the very same method, one can calculate the density distribution according to the accurate formula for all velocities corresponding to elliptical orbits. It can be concluded from the Table, which combines α and η , for different v_1 , that a result is obtained of the same order of magnitude, because the change in v_1 points only slightly to the given dependence.

We thus arrive at the conclusion that it is absolutely possible to represent the observed density distribution in the zodiacal light both in the elliptical plane, and in the direction which is normal to it, if it is assumed that particles of the zodiacal light are ejected into space by asteroids which are continuously bombarded by meteors which are flying into them. There is no doubt that interplanetary space is enriched by a similar mechanism. The question only arises as to whether this mechanism can provide the appropriate relative ejection velocity, which must exceed the orbital velocity of the asteroids and, in addition, which must be contained within rather narrow limits.

/100

The weak aspect of the theory lies here. It is fully possible that the purely dynamic considerations which have been developed in this work are not completely adequate, and must be supplemented by introducing radiative deceleration, which forces the particles to approach the sun gradually at very small distances. In this way, the necessity of assuming large ejection velocities is avoided. However, it is first necessary to examine the general properties for the motion of a dust-like particle with respect to the sun, which is subjected not only to the force of gravity, but also to light repulsion.

CHAPTER 7

Motion of Cosmic Dust in Interplanetary Space

1. The problem of the motion of cosmic dust in interplanetary space is very important in cosmogony, and is also important for the formulation of a theory on the structure of the interplanetary dust medium. Let us examine the problem of a change in the elliptical orbital elements of an individual dust particle moving under the influence of solar attraction and light repulsion. It is assumed that the dust particle is neutral, and is not subject to the influence of an electric or magnetic field. It should be noted that, in fact, we are not at all interested in the motion of a dust particle along an orbit, but only in the secular change in its elements. Evidently, it is not possible to investigate the elements of an elliptical orbit ω , Ω , T , - i.e., the distance of the perihelion from the node, the longitude of the node, and the epoch. In addition, because both forces acting on the dust particle occur in the same plane with the velocity vector, the orbit inclination apparently remains unchanged, and therefore does not require an investigation.

/101

We shall thus confine ourselves to investigating the secular changes in the elliptical elements a and e , the semimajor axis, and the eccentricity.

2. Let the origin be located at the sun, and let the particle move in the X, Y plane. In a rectangular coordinate system, we have two equations of motion:

$$m \frac{d^2 x}{dt^2} = X_1 + X_2; \quad m \frac{d^2 y}{dt^2} = Y_1 + Y_2,$$

where X_1, Y_1 are the components of the attractive force and, F, X_2, Y_2 are the components of the light repulsion force F_2 .

/102

The following apparently holds

$$X_1 = -\frac{k^2 M m x}{r^3}; \quad Y_1 = -\frac{k^2 M m y}{r^3},$$

where M, m are the mass of the sun and the particle, and r is the radius vector.

We should note that the direction of the vector F_2 and the radius vector r comprise a small angle γ , which represents the

angle of annual aberration

$$\gamma = \frac{v}{c} \sin u,$$

where u is the angle between the direction of the particle velocity v and the direction toward the sun. For the earth, the maximum value of γ is

$$\gamma_{max} = \frac{v}{c} = \frac{1}{10\,000} = 20.''5.$$

It is apparent that

$$X_2 = F_2 \cos \gamma \frac{x}{r} + F_2 \sin \gamma \frac{y}{r}; \quad Y_2 = F_2 \cos \gamma \frac{y}{r} - F_2 \sin \gamma \frac{x}{r}.$$

The magnitude of the repulsion force F_2 is determined by the amount of energy falling on the particle from the sun. If ρ is the particle radius, and E_0 is the amount of energy falling per $1 \text{ cm}^2/\text{sec}$ at a distance of r_0 from the sun, then

/103

$$F_2 = \frac{\pi \rho^2 r_0^2 E_0}{c r^2} = \frac{B}{r^2},$$

is the force of light repulsion for an absolutely black particle.

With sufficient accuracy, we can set

$$\cos \gamma = 1; \quad \sin \gamma = \gamma.$$

Since light repulsion - similarly to gravitational attraction - changes in a manner which is inversely proportional to the square of the distance, it is expedient to introduce a "reduced" mass of the sun M'

$$M' = M - \frac{\pi \rho^2 r_0^2 E_0}{c k^2 m} = M - \frac{3}{4} \frac{r_0^2 E_0}{c k^2 \rho \delta}$$

(δ is the particle density).

Therefore, the equations of motion

$$m \frac{d^2 x}{dt^2} = - \frac{k^2 M m x}{r^3} + \frac{B x}{r^3} + \frac{B y v \sin u}{r^3 c};$$

$$m \frac{d^2 y}{dt^2} = - \frac{k^2 M m y}{r^3} + \frac{B y}{r^3} - \frac{B x v \sin u}{r^3 c}.$$

can be transformed to the form

$$m \frac{d^2 x}{dt^2} = - \frac{k^2 M' m x}{r^3} + \frac{B y v \sin u}{r^2 c},$$

$$m \frac{d^2 y}{dt^2} = - \frac{k^2 M' m y}{r^3} - \frac{B x v \sin u}{r^2 c}, \quad B = \frac{\pi e^2 r_0^2 E_0}{c}.$$

We obtain the following combination from these equations:

$$\frac{d}{dt} m \left(y \frac{dx}{dt} - x \frac{dy}{dt} \right) = \frac{B v \sin u}{rc}$$

$$\frac{1}{2} \frac{dm}{dt} \left[\left(\frac{dx}{dt} \right)^2 + \left(\frac{dy}{dt} \right)^2 \right] = - \frac{k^2 M' m}{r^3} \left(x \frac{dx}{dt} + y \frac{dy}{dt} \right) +$$

$$+ \frac{B}{r^3} \frac{v}{c} \sin u \left(y \frac{dx}{dt} - x \frac{dy}{dt} \right)$$

or, changing to polar coordinates, we have

/104

$$x = r \cos \varphi, \quad y = r \sin \varphi$$

$$\frac{d}{dt} \left(m r^2 \frac{d\varphi}{dt} \right) = - \frac{B v \sin u}{rc}$$

$$\frac{1}{2} \frac{d}{dt} (m v^2) = - \frac{k^2 M' m}{r^2} \frac{dr}{dt} - \frac{B v}{r^2 c} \sin u \frac{d\varphi}{dt}.$$

The angle u between the tangent and the radius vector is given by the relationship

$$\operatorname{tg} u = - \frac{r d\varphi}{dr},$$

from which it follows that

$$\sin u = \frac{r}{v} \frac{d\varphi}{dt}.$$

Consequently,

$$v \sin u = - r \frac{d\varphi}{dt}.$$

Therefore,

$$\frac{d}{dt} \left(m r^2 \frac{d\varphi}{dt} \right) = - \frac{B}{c} \frac{d\varphi}{dt}; \quad (11)$$

$$\frac{1}{2} \frac{d}{dt} (m v^2) = - \frac{k^2 M' m}{r^2} \frac{dr}{dt} - \frac{B}{c} \left(\frac{d\varphi}{dt} \right)^2. \quad (12)$$

The first equation can be readily integrated

$$mr^2 \frac{d\varphi}{dt} + \frac{B}{c} \varphi = \text{const.}$$

At an initial moment of time, it is always possible to set

$$\varphi_0 = 0.$$

Therefore

$$r^2 \frac{d\varphi}{dt} + \frac{B\varphi}{cm} = \left(r^2 \frac{d\varphi}{dt} \right)_0.$$

The first term of this expression represents the areal velocity of the particle. This areal velocity continuously decreases /105 in the same measure that the second term increases - namely, it is proportional to φ . For each revolution along the orbit, no matter what its dimensions, the areal velocity decreases by one and the same quantity

$$\frac{2\pi}{m} \frac{B}{c}.$$

If we define the initial areal velocity as

$$\left(r^2 \frac{d\varphi}{dt} \right)_0 = \sqrt{k^2 M' a_0 (1 - e_0^2)},$$

we can then find the amount of revolutions which the dust particle performs along its orbit, which turns gradually in a spiral-like manner until it falls into the sun. In fact, as the sun is approached

$$r^2 \frac{d\varphi}{dt} \rightarrow 0.$$

Therefore,

$$\frac{2\pi n B}{mc} = \sqrt{k^2 M' a_0 (1 - e_0^2)},$$

from which it follows that

$$n = \frac{\sqrt{k^2 M' a_0 (1 - e_0^2)} mc^2}{2\pi^2 \rho^2 r_0^2 E_0},$$

or

$$n = \frac{\sqrt{k^2 M' a_0 (1 - e_0^2)}}{4\pi r_0^2 E_0} \cdot \frac{8}{3} \rho \delta c^2$$

We have

$$4\pi r_0^2 E_0 = 4 \cdot 10^{33} \text{ erg/sec} - \text{total radiation of the sun}$$

$$c^2 = 9 \cdot 10^{20} \text{ cm}^2/\text{sec}^2; \quad M' = M - \frac{3}{16} \cdot \frac{4\pi r_0^2 E_0}{\pi c k^2 \rho \delta}.$$

Let us set, for example,

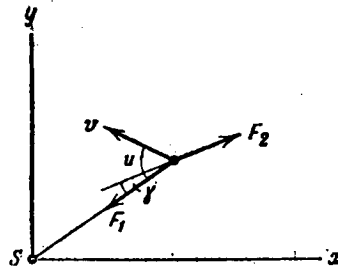
/106

$$\delta = 1; \quad a_0 = 1.5 \cdot 10^{13} \text{ sec};^* \quad e_0 = 0;$$

$$k^2 = 6.7 \cdot 10^{-8};$$

$$M' = 2 \cdot 10^{33} \left(1 - \frac{10^{-4}}{1.7\rho} \right).$$

If $1.7\rho = 10^{-4} \text{ cm}$ - i.e., the particle diameter is 1μ - then $M' = 0$, and the attractive force is counterbalanced by the repulsion.



/102

Figure 5

Motion of Meteoric Dust in the Solar System.

We shall assume that the particle diameter is at least several tens of microns.

/106

For $2\rho = 10\mu$, the order of magnitude for M' remains the same as for M , so that for approximate calculations it can be assumed that

$$M = M'.$$

We have

$$n = 2.7 \cdot 10^7 \rho \delta.$$

If $\delta = 1$, $\rho = 10^{-3}$ is the smallest assumed quantity, then

* Translator's note: "sec" incorrect in original. Should be "cm" (i.e., number of cms. in AU).

$$n = 2.7 \cdot 10^4.$$

In general, even a small particle must perform hundreds of thousands of rotations in order to fall into the sun, beginning with the distance of the terrestrial orbit.

We shall introduce the elliptical elements a , e as new variables in equations (11) and (12). We have

$$\frac{d}{dt} m \sqrt{k^2 M' a (1 - e^2)} = - \frac{B}{c} \frac{d\varphi}{dt}$$

and

$$\frac{d}{dt} \left(\frac{mk^2 M'}{2a} \right) = \frac{B}{c} \left(\frac{d\varphi}{dt} \right)^2.$$

The quantity $\frac{d\varphi}{dt}$ represents the instantaneous angular velocity which decreases periodically throughout a revolution along the orbit, and changes in a secular way when the shape and dimensions of the orbit change. /107

Since we are only interested in the secular changes a and e , we can perform preliminary integration with respect to the period of rotation T .

For example, we have

$$\int_0^T \frac{d}{dt} \left(\frac{k^2 M'}{2a} \right) dt = \int_0^T \frac{B}{mc} \left(\frac{d\varphi}{dt} \right)^2 dt.$$

It was shown above that a very large number of rotations along the orbit is necessary for a significant change in a . The magnitude of a does not change significantly throughout the period of one rotation T . Therefore,

$$\frac{d}{dt} \left(\frac{k^2 M'}{2a} \right) = \frac{1}{T} \int_0^T \left(\frac{d\varphi}{dt} \right)^2 dt \frac{B}{mc}$$

and similarly for the second equation

$$\frac{d}{dt} \sqrt{k^2 M' a (1 - e^2)} = - \frac{B}{mc} \frac{1}{T} \int_0^T \left(\frac{d\varphi}{dt} \right) dt.$$

We have

$$\frac{1}{T} \int_0^T \left(\frac{d\varphi}{dt} \right)^2 dt = \frac{\sqrt{k^2 M' a (1 - e^2)}}{T} \int_0^{2\pi} \frac{d\varphi}{r^2},$$

because

$$r^2 \frac{d\varphi}{dt} = \sqrt{k^2 M' a (1 - e^2)}.$$

But

$$r = \frac{a(1 - e^2)}{1 + e \cos(\varphi - \varphi_0)}.$$

and therefore

$$\frac{1}{T} \int_0^T \left(\frac{d\varphi}{dt} \right)^2 dt = \frac{\sqrt{k^2 M' a (1 - e^2)}}{T a^3 (1 - e^2)^2} \int_0^{2\pi} [1 + e \cos(\varphi - \varphi_0)]^2 d\varphi.$$

After simple integration, we obtain

/108

$$\frac{1}{T} \int_0^T \left(\frac{d\varphi}{dt} \right)^2 dt = \frac{\sqrt{k^2 M' a (1 - e^2)}}{T a^3 (1 - e^2)^2} \cdot 2\pi \left(1 + \frac{e^2}{2} \right).$$

But

$$\frac{2\pi}{T} = \frac{\sqrt{k^2 M'}}{a^{3/2}}.$$

Consequently,

$$\frac{1}{T} \int_0^T \left(\frac{d\varphi}{dt} \right)^2 dt = \frac{k^2 M'}{a^3} \cdot \frac{\left(1 + \frac{e^2}{2} \right)}{(1 - e^2)^{5/2}}.$$

and

$$\frac{da}{dt} = -\frac{2B}{mca} \cdot \frac{\left(1 + \frac{e^2}{2} \right)}{(1 - e^2)^{5/2}}. \quad (13)$$

In the same way we find

$$\frac{d}{dt} \sqrt{k^2 M' a (1 - e^2)} = -\frac{B}{mc} \cdot \frac{\sqrt{k^2 M'}}{a^{3/2}},$$

from which we obtain the following, taking (13) into account:

$$\frac{de}{dt} = -\frac{3}{2} \frac{Be}{mca^2} \cdot \frac{1}{(1 - e^2)^{1/2}}. \quad (14)$$

The element of time must be first excluded in order to

integrate equations (13) and (14). We obtain

$$\frac{de}{da} = \frac{3}{4} \frac{1}{a} \cdot \frac{e}{1 + \frac{e^2}{2}} (1 - e^2),$$

from which it follows that

$$-\frac{3}{4} \int \frac{da}{a} = \int \frac{de}{e} \frac{\left(1 + \frac{e^2}{2}\right)}{(1 - e^2)}.$$

Performing elementary integration, we obtain

/109

$$\frac{3}{4} \ln \frac{a}{a_0} = \ln \left(\frac{e}{e_0} \frac{(1 - e_0^2)^{3/4}}{(1 - e^2)^{3/4}} \right)$$

or

$$\frac{a}{a_0} = \left(\frac{e}{e_0} \right)^{4/3} \frac{1 - e_0^2}{1 - e^2}. \quad (15)$$

It can be seen from equation (14) that the eccentricity of the orbit must continuously decrease as a result of radiative deceleration. If $e = 0$ initially, then $\frac{de}{dt} = 0$, and the orbit always remains circular. Expression (15) shows that each orbit is initially changed into a circular orbit with any eccentricity only for $a = 0$. Particles moving along elliptical orbits exist for a somewhat shorter period of time than do particles describing circular orbits.

It is also interesting to note that the determined dependence between a and e expresses a general law of motion in the presence of radiative deceleration, and does not depend on individual properties of the moving body. It does not matter whether an asteroid or an insignificant dust particle moves in the field of radiation and gravitation; the relationship between the elements a and e is the same for them. The only difference lies in the fact that the asteroid orbit changes incredibly slowly, while the orbit of the dust particle changes comparatively rapidly.

If we know the dependence between a and e , we can integrate equations (13) and (14). For example:

$$\frac{de}{dt} = -\frac{3}{2} \frac{Be}{mc} \frac{e_0^{1/3} (1 - e^2)^2}{(1 - e^2)^2 a_0^2 e^{1/3} (1 - e_0^2)^2}$$

or

$$\frac{de}{dt} = -K \frac{(1-e^2)^{3/2}}{a^2},$$

where

/110

$$K = \frac{3}{2} \frac{B}{mc} \frac{e_0^{3/2}}{a_0^2} \cdot \frac{1}{(1-e_0^2)^{3/2}}; \quad B = \frac{\pi \rho^2 r_0^2 E_0}{c}.$$

It thus follows that

$$\int_{e_0}^e \frac{de e \sqrt{e^2}}{(1-e^2) \sqrt{1-e^2}} = K(t-t_0)$$

or, setting $e^2 = x$, we have

$$\int_{e_0^2}^{e^2} \frac{dx \sqrt{x}}{(1-x) \sqrt{1-x}} = 2K(t-t_0).$$

It is not at all difficult to calculate the latter integral by the numerical method. As a result of the calculations, we have

$$2K(t-t_0).$$

TABLE 13

$e \backslash e_0$	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1
0.8	1.030	—	—	—	—	—	—	—	—
0.7	1.472	0.442	—	—	—	—	—	—	—
0.6	1.699	0.669	0.227	—	—	—	—	—	—
0.5	1.827	0.797	0.355	0.128	—	—	—	—	—
0.4	1.902	0.872	0.430	0.203	0.0749	—	—	—	—
0.3	1.945	0.916	0.474	0.246	0.1177	0.0428	—	—	—
0.2	1.965	0.936	0.494	0.266	0.1398	0.0649	0.0221	—	—
0.1	1.974	0.945	0.503	0.275	0.1488	0.0739	0.0311	0.0096	—
0.0	1.976	0.947	0.505	0.277	0.1504	0.0755	0.0327	0.0106	0.00156

We can find the corresponding a for initial values of a_0 , e_0 from the relationship

$$\frac{a}{a_0} = \left(\frac{e}{e_0} \right)^{1/2} \cdot \frac{1-e_0^2}{1-e^2}$$

Let us assume that $a = 1.5 \cdot 10^{13}$ cm is the distance from the earth to the sun; $4\pi r_0^2 E_0 = 4 \cdot 10^{33}$ erg/sec; $\delta = 1$; $\rho = 10^{-3}$ cm.

/112

TABLE 14

/111

$e \backslash e_0$	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1
0.8	$4.42 \cdot 10^3$ 0.449	— —	— —	— —	— —	— —	— —	— —	— —
0.7	6.31 0.265	$0.985 \cdot 10^3$ 0.591	— —	— —	— —	— —	— —	— —	— —
0.6	7.27 0.172	1.414 0.383	$1.368 \cdot 10^3$ 0.648	— —	— —	— —	— —	— —	— —
0.5	7.84 0.115	1.683 0.237	2.14 0.432	$1.831 \cdot 10^3$ 0.668	— —	— —	— —	— —	— —
0.4	8.17 0.0764	1.846 0.1696	2.59 0.287	2.91 0.444	$2.40 \cdot 10^3$ 0.650	— —	— —	— —	— —
0.3	8.34 0.0479	1.952 0.1068	2.86 0.1805	3.53 0.278	3.76 0.417	$3.27 \cdot 10^3$ 0.628	— —	— —	— —
0.2	8.43 0.0265	1.977 0.0591	2.98 0.100	3.82 0.154	4.40 0.2305	4.65 0.346	$3.67 \cdot 10^3$ 0.551	— —	— —
0.1	8.46 0.0103	1.994 0.0227	3.03 0.0385	3.94 0.0593	4.68 0.0875	5.32 0.134	5.34 0.212	$5.41 \cdot 10^3$ 0.383	— —
0.0	$8.48 \cdot 10^3$ 0.000	$2.00 \cdot 10^3$ 0.00	3.045 0.000	3.97 0.000	4.71 0.000	5.47 0.000	5.69 0.000	6.42 0.000	$6.84 \cdot 10$ 0.000

For these data, we have the following values of K:

/112

e_0 K	0.9	0.8	0.7	0.6	0.5
	$1.17 \cdot 10^{-3}$	$2.36 \cdot 10^{-4}$	$8.32 \cdot 10^{-5}$	$3.48 \cdot 10^{-5}$	$1.57 \cdot 10^{-5}$

e_0 K	0.4	0.3	0.2	0.1
	$8.18 \cdot 10^{-6}$	$2.98 \cdot 10^{-6}$	$8.27 \cdot 10^{-7}$	$2.44 \cdot 10^{-7}$

The first row of each column in Table 14 gives the time interval $t - t_0$ expressed in years, during which the orbit is changed from e_0 to e . The second row of the same columns presents the corresponding value of $\frac{a}{a_0} - i.e.$, the relationship between the final and initial semimajor axis.

The problem can be completely solved for any initial value of (a_0, e_0) , and for any properties of the moving particle by means of this Table.

Let us set, for example,

$$\delta=3; \rho=1 \text{ cm}; a_0=2; e_0=0.5.$$

It is necessary to find the time required for a particle to fall into the sun ($a = 0$). We have

$$t-t_0=4.71 \cdot 10^3 \cdot 3 \cdot 1000 \cdot 2^2=5.66 \cdot 10^7 \text{ years}$$

Similarly, the time required for a decrease in a_0 to 0.4 of its value will be

$$t-t_0=3.8 \cdot 10^3 \cdot 3 \cdot 1000 \cdot 2^2=4.56 \cdot 10^7 \text{ years}$$

It is interesting to note that for the same semimajor axis the time required for a particle to fall into the sun is much less for small eccentricities. Particles moving along extended orbits approach the sun more rapidly.

/113

In the special case of a circular orbit, the radiative deceleration can be taken into account in an extremely simple manner.

We already know that the orbit which is initially circular always remains circular. Therefore, for any moment in time we have

$$\frac{mv^2}{r} = \frac{k^2 M' m}{r^2}$$

M' is the reduced solar mass, i.e.,

$$v^2 = \frac{k^2 M'}{r}$$

The change in the momentum will be

$$d(mv) = \gamma \frac{B dt}{r^2}, \quad \gamma = \frac{v}{c}$$

As a result of the change, v also changes as well as r

$$v dv = - \frac{k^2 M'}{2r^2} dr$$

Consequently, we have

$$- \frac{k^2 M' dr}{2r^2 v} = \frac{v B dt}{c \cdot m r^2},$$

from which it follows that

$$\frac{dr}{dt} = -\frac{2v^2}{c} \frac{B}{mk^2M'}$$

or

$$\frac{dr}{dt} = -\frac{3}{2} \frac{r_0^2 E_0}{\rho \delta c^2 r} \quad (16)$$

This means that the rate at which the radius of the circular orbit is reduced is inversely proportional to the quantity $r\rho\delta$

$$\frac{dr}{dt} = -\frac{10^{13}}{6\pi\rho\delta r} = -\frac{A}{\rho\delta r}$$

Thus, the time τ required to fall on the sun from the distance r_0 is /114

$$\tau = \frac{\rho\delta r_0^3}{2A} \text{ sec} = \frac{\rho\delta r_0^3 \cdot 10^{-17}}{6A} \text{ years}$$

If we assume $\delta = 1$ and r_0 is expressed in astronomical units, then

$$\tau = 6.5 \cdot 10^6 \rho r_0^3 \text{ years}$$

If $r_0 = 1$ and $\rho = 10\mu$, then $\tau = 6500$ years.

It is interesting to note that the force of solar attraction does not appear in expression (16).

3. It was assumed above that particles moving in the solar system are neutral, and are subjected only to the force of gravity and to light repulsion. In actuality, it can be expected that they have a certain electric charge, due to the photoeffect produced by hard quanta emitted by the sun. In such cases, the over-all magnetic and electric field of the sun will influence them. However, it must be noted that the problem of the motion of an electric charge in the solar system does not have the requisite element of certainty, which is characteristic of problems in celestial mechanics.

Let us investigate a particle having the cross-section $\pi\rho^2$ at the distance r from the sun. Light quanta fall on the particle, including those which can knock out an electron and impart a positive charge to the particle. On the basis of Einstein's equations, the energy which is necessary for this will be

$$\begin{aligned} h\nu_1 &= P & (\text{for the first electron}) \\ h\nu_2 &= P + \frac{e^2}{\rho} & (\text{for the second electron}) \end{aligned}$$

$$h\nu_3 = P - \frac{2e^2}{\rho} \quad (\text{for the third electron})$$

.....

.....

$$h\nu_{k+1} = P - \frac{ke^2}{\rho} \quad (\text{for the } k\text{th electron})$$

The number of corresponding quanta per one second is

/115

$$n_k = \frac{\pi \rho^2 r_0^2}{r^2} \int_v^\infty \frac{E_v^0 dv}{h\nu}$$

The expression $4\pi r_0^2 E_v^0 dv$ represents the general radiation of the sun in the frequency interval from ν to $\nu + dv$.

It is known that

$$4\pi r_0^2 \int_0^\infty E_v^0 dv = 4 \cdot 10^{33} \text{ erg/sec.}$$

Let us assume that at a certain initial moment the particle is neutral. The first electron will be ejected from it after

$\frac{1s}{n_1}$ sec; the second electron - $\frac{1s}{n_2}$ sec. after this; the third electron - $\frac{1s}{n_3}$ sec. after this, etc.

Consequently, the electrization rate will be $n_1, n_2, \dots$

The value of n decreases as the charge increases.

Along with this, we must investigate the rate at which free electrons recombine with the particles. Under stationary conditions, both rates must be identical, and this determines the final particle charge.

Let us assume that there are N_e free electrons per unit of volume in space; these electrons are characterized by the kinetic temperature T_e , according to the corresponding Maxwell velocity distribution. Let dN_e be the number of electrons per unit of volume, whose velocities are contained between v and $v + dv$. The number of electrons in the solid angle $d\omega$, which have this velocity and which encounter a particle in one second, will be

$$\frac{\pi \rho^2 v dN_e d\omega}{4\pi}$$

The total number of electrons is

/116

$$n_1' = \pi p^2 \int_0^{\infty} v dN_e.$$

According to Maxwell

$$dN_e = C v^2 e^{-\frac{mv^2}{2kT_e}} dv,$$

while

$$N_e = C \int_0^{\infty} v^2 e^{-\frac{mv^2}{2kT_e}} dv.$$

Therefore,

$$n_1' = \pi p^2 \frac{\int_0^{\infty} v^3 e^{-\frac{1}{2} \frac{mv^2}{kT_e}} dv N_e}{\int_0^{\infty} v^2 e^{-\frac{1}{2} \frac{mv^2}{kT_e}} dv}.$$

We obtain the same expressions for recombination in the case of charged particles, with the only difference being that the effective cross-section σ of a charged particle is greater than its actual cross-section, depending on velocity and charge.

Using $\sigma_k(v)$ to designate the effective cross-section with the charge ke , we can represent the number of electrons falling on a particle per one second by the similar expression

$$n_k' = \frac{\int_0^{\infty} \sigma_k(v) v^3 e^{-\frac{1}{2} \frac{mv^2}{kT_e}} dv N_e}{\int_0^{\infty} v^2 e^{-\frac{1}{2} \frac{mv^2}{kT_e}} dv}.$$

A stationary state is achieved under the condition:

$$n_k = n_k',$$

i.e.,

/117

$$\frac{\pi p^2 r_0^2}{r^2} \int_{v_k}^{\infty} \frac{E_v^0 dv}{h\nu} = N_e \frac{\int_0^{\infty} \sigma_k(v) v^3 e^{-\frac{1}{2} \frac{mv^2}{kT_e}} dv}{\int_0^{\infty} v^2 e^{-\frac{1}{2} \frac{mv^2}{kT_e}} dv}.$$

The left part of the equation decreases with the charge, since v_k increases. On the other hand, the right part increases, due to an increase in $\sigma_k(v)$. Under stationary conditions, both

parts must be identical.

Let us determine the effective cross-section of a particle $\sigma_k(v)$ with the charge ke . Let the electron velocity at a large distance from the particle be v . From the integral of kinetic energies

$$\frac{mv^2}{2} = \frac{ke^2}{r} + \frac{ke^2}{a}$$

we can find the actual semiaxis of the hyperbolic orbit, namely

$$a = \frac{2ke^2}{mv^2}.$$

The angle between the asymptote and the axis of the hyperbola α is connected to the eccentricity e by the relationship

$$e = \sec \alpha.$$

The effective radius of the particle is, apparently, the length of the perpendicular dropped from its center to the asymptote of the hyperbola. Assuming that the distance of the perihelion q equals the real particle radius ρ , we find

$$\rho_{eff} = (a + \rho) \sin \alpha$$

or

$$\rho_{eff} = \left(\frac{2ke^2}{mv^2} + \rho \right) \frac{\sqrt{(a + \rho)^2 - a^2}}{a + \rho}.$$

Consequently, the effective cross-section is

/118

$$\rho_{eff} = \rho \left(1 + \frac{4ke^2}{\rho mv^2} \right)^{\frac{1}{2}}.$$

Let us now turn to calculating the particle charge ke .

According to Spitzer (Ref. 39), we can assume that the photo-electric effect occurs for particles of cosmic dust when they are bombarded by light quanta having a wavelength which is less than 2,000 Å. It can be assumed that each such quantum can eject one electron. We shall examine the case below, in which the ejection probability for electrons does not exceed 0.001 during this bombardment.

For purposes of definition, let us assume a particle having the radius $\rho = 10^{-3}$ cm. After substituting numerical data in the original equation

$$h\nu_k = h\nu_0 + \frac{ke^2}{\rho}$$

we find:

$$\nu_k = 1.5 \cdot 10^{15} + k \cdot 3 \cdot 10^{10}.$$

In order to calculate the number of quanta per one second which ionize the particle

$$n_k = \frac{\pi \rho^2 r_0^2}{r^2} \int_{\nu_k}^{\infty} \frac{E_\nu^0 d\nu}{h\nu},$$

for purposes of simplicity, we shall assume an energy distribution in the solar spectrum according to the formula of Wien

$$E_\nu^0 \propto \nu^3 e^{-\frac{h\nu}{\chi T}}.$$

Introducing the new variable

$$\alpha = \frac{h\nu}{\chi T}, \quad \alpha_k = \frac{h\nu_k}{\chi T}$$

we find the following for the number of quanta per 1 second

/119

$$n_k = \frac{\rho^2 \cdot 10^{13}}{r^2 \chi T} \frac{\int_{\alpha_k}^{\infty} \alpha^2 e^{-\alpha} d\alpha}{\int_0^{\infty} \alpha^3 e^{-\alpha} d\alpha}.$$

Taking the fact into account that

$$\int_0^{\infty} \alpha^3 e^{-\alpha} d\alpha = 6,$$

we find

$$n_k = \frac{\rho^2}{r^2 (\chi T)} \frac{10^{13}}{6} \cdot e^{-\alpha_k} (\alpha_k^2 + 2\alpha_k + 2),$$

$$\alpha_k = \frac{h\nu_k}{\chi T}.$$

We shall assume that the effective temperature of the sun is 6000°, and also that this temperature characterizes the energy distribution in the remote ultraviolet portion of the solar spectrum. As far as we can determine, there is no basis for assuming that there is a significant ultraviolet radiation excess, with the possible exception of individual regions of the solar disc which are insignificant in size.

The preceding expression for the distance from the earth to the sun

$$r = 1.5 \cdot 10^{13} \text{ cm}$$

takes the following form

$$n_k = 0.905 \cdot 10^{12} e^{-\alpha_k} (\alpha_k^2 + 2\alpha_k + 2),$$

while

$$\alpha_k = \frac{h\nu_k}{kT} = 12 + k \cdot 2.4 \cdot 10^{-4}.$$

We can now calculate

$$n_k' = \frac{\int_0^\infty v^3 e^{-\frac{1}{2} \frac{mv^2}{kT}} dv}{\int_0^\infty v^2 e^{-\frac{1}{2} \frac{mv^2}{kT}} dv} \cdot N_e$$

or

/120

$$n_k' = \frac{\int_0^\infty v^3 e^{-\frac{1}{2} \frac{mv^2}{kT}} dv + \frac{4ke^2}{\rho m} \int_0^\infty v e^{-\frac{1}{2} \frac{mv^2}{kT}} dv}{\int_0^\infty v^2 e^{-\frac{1}{2} \frac{mv^2}{kT}} dv} N_e \pi \rho^2.$$

This expression can be readily calculated with the aid of elementary functions. Taking the fact into account that

$$\int_0^\infty \beta^2 e^{-\beta^2} d\beta = \frac{\sqrt{\pi}}{4},$$

after various transformations, we can determine the following for the number of recombinations per one second

$$n_k' = 8 \left(\frac{kT}{m} \right)^{\frac{1}{2}} \frac{N_e \pi \rho^2}{\sqrt{\pi}} \left[1 + \frac{3ke^2}{\rho m (kT)} \right].$$

Making numerical substitutions as was indicated above, and taking the fact into account that

$$\sigma_{eff} = \pi \rho^2 (1 + 4.9 \cdot 10^{-4} k),$$

we find

$$n_k' = 425 N_e (1 + 4.9 \cdot 10^{-4} k).$$

The equation of balance assumes the form

$$0.905 \cdot 10^{12} e^{-\alpha_k} (\alpha_k^2 + 2\alpha_k + 2) = 425 N_e (1 + 4.9 \cdot 10^{-4} k),$$

while

$$\alpha_k = 12 + 2.4 \cdot 10^{-4} k$$

Assuming that

$$x = k \cdot 10^{-5},$$

we can change the preceding equation to the form

$$2.23 \cdot 10^6 e^{-10 \cdot 42 x} (3.39 x^2 + 3.67 x + 1) = N_e (1 + 49 x).$$

The solution of this equation, which can be found by the trial method, leads to the following approximate values of k for the different electron density N_e

/121

N_e	1	10	100
k	$5.4 \cdot 10^4$	$4.4 \cdot 10^4$	$3.4 \cdot 10^4$

We must remember that it has been assumed that each quantum having a hardness greater than 2,000 Å can eject an electron when falling on a particle.

If it is assumed that only one electron is ejected per each thousand quanta, then the positive charge of the particle will be somewhat less, namely:

N_e	1	10	100
k	$2.5 \cdot 10^4$	$1.6 \cdot 10^4$	$0.8 \cdot 10^4$

The corresponding charge in volts, calculated according to the formula

$$\frac{ke^2}{\rho} = \frac{eV}{300},$$

is given below:

N_e	1	10	100
V (prob. of 1).	7.2	5.9	4.6
V (prob. of 10^{-3}).	3.3	2.1	1.1

It is interesting to note that a charge is obtained of the same order of magnitude for very great changes in the number of free electrons N_e and the probability of the photoeffect. It can be assumed that N_e equals approximately 1 in interstellar space. In all probability, N_e is larger in the solar system, but it cannot exceed 100, since for electron scattering power the brightness of the night sky will be much greater in this case and will be independent of the ecliptical latitude - which is not observed in actuality. It can therefore be assumed that the possible positive charge does not exceed 2-3 volts even for the largest meteoric particles. These data pertain to the distance from the sun for one astronomical unit. As can be seen from the table, the charge must be of the same order of magnitude at other distances. For such a charge, the particle motion must be determined almost exclusively by the force of gravity and by normal light repulsion. Nothing definite is known about the presence of an external magnetic field of the sun, and there is some doubt as to whether it even exists at all. On the other hand, in order that the effect of an electric force on a particle can be compared with the force of gravity, it is necessary that the charge of the sun be on the order of 10^{15} volts, which is also completely improbable. Thus, all conclusions regarding the motion of a particle in interplanetary space remain in force. /122

4. An investigation was made above on the time required for a meteoric particle to fall into the sun or, more precisely, for the semimajor axis of the orbit to decrease to zero. We should note that during the flight to the sun an actual decrease in the amount of angular momentum connected with the particle cannot take place. We have seen that, no matter what the nature of the initial orbit for the meteoric particle, it approaches the sun along a circular spiral, when the particle - performing several consecutive revolutions - only very slowly decreases its distance to the sun. Thus, the particle is gradually heated, and it begins to decrease its size at a certain distance. This leads to the fact that the reduced mass of the sun also decreases more and more rapidly, and finally reaches zero. However, it can be shown - as was done by Jeans (Ref. 40) - that at each given moment the product of the semimajor axis for the osculating orbit of the particle by the reduced mass of the sun remains constant, i. e., /123

$$aM' = \text{const.}$$

The semimajor axis increases as M' decreases. This very quickly balances the approach to the sun as a result of radiative deceleration, and then leads to a more or less rapid departure of the particle. Thus, the fall can actually take place only in the case of a cold sun,

but in this case the sun would not be in a position to evidence radial repulsion.

We should point out that the reduced mass of the sun decreases significantly only when the particles are very small in size, on the order of several microns. Therefore, as it begins to decrease, the particle actually volatilizes and - as it is repulsed from the sun - it changes into a cloud of ionized gases which rapidly dissipates into space.

Since it is very probable that this process takes place quite rapidly, in spite of the very slow approach to the sun, there is no necessity of investigating the evolution of the particle's orbit at this stage of its existence.

5. Significant importance can only be attributed to the fact that meteoric particles, changing into ionized gases, must enrich interplanetary space with free electrons which have considerable scattering ability. It can actually be shown that even hydrogen atoms, which have a significant ionization potential, must lose their electrons throughout all of space in the solar system.

Let us examine the basic ionization equation for interstellar space (Ref. 41):

$$\frac{N'' N_e}{N'} = \frac{(2\pi me)^{3/2} 2q''}{h^3 q'} (kT)^{1/2} e^{-\frac{J}{kT}} \sqrt{\frac{T_{eff}}{T}} W e^{-\tau},$$

N' , N'' , N_e represents the amount of neutral atoms in the basic state, and ions and electrons per unit of volume; q' , q'' - statistical weights of the basic state of an atom and an ion; T , T_{eff} - effective temperature of adjacent stars and kinetic temperature of free electrons; J - ionization potential; τ - optical thickness at the distance S from the star, at which the state of the given gas is investigated; W - radiation dilution factor, namely

/124

$$W = \frac{R^2}{S^2}.$$

R designates the radius of the star, while it is assumed that

$$R \ll S.$$

We can assume that

$$N = N' + N''; N'' = xN; N' = (1-x)N.$$

In the presence of only first order ionization, the number

of electrons must equal the number of ions, and therefore we have

$$N_e = xN.$$

Substituting these expressions in the preceding formula and inserting the values of certain constants, we find

$$\frac{x^2}{1-x} N = C_1 \cdot \frac{1}{S^2} \cdot e^{-\tau},$$

while

$$C_1 = 10^{-0.51 - 0.12 \frac{q''}{q'}} \sqrt{\frac{T_{eff}}{T}} T^{0.1} R^2,$$

where

$$\tau = \frac{5040}{T}.$$

Within the limits of the solar system, it can be naturally assumed that

$$\tau = 0.$$

In the case of hydrogen atoms in the solar system ($R = 1$, $x = 13.56$ volts, $T = T_{eff} = 6000^\circ$) we have

/125

$$\frac{Nx^2S^2}{1-x} = 6.12 \cdot 10^{-5}.$$

For the degree of hydrogen ionization $x = 0.5$ and 0.9 , we obtain

	$x = 0.5$	$x = 0.9$
$N = 1$	$S = 2300$ astr. unit	$S = 560$ astr. unit
10	740	179
100	230	56
1000	74	18

As was pointed out above, for $N = 1000$ the entire brightness of the night sky can only be caused by electron scattering. Actually, if we take the general expression (Ref. 42) for the brightness of an electron cloud

$$J = \mu L \int_0^\infty \frac{N f(\theta) dR}{r^2}$$

(L - amount of light emitted by the sun per unit of solid angle, i.e., per one steradian; $\mu f(\theta)$ - scattering ability in the direction θ between incident and scattered rays, calculated for one electron) and if we take the fact into account that in units

of the terrestrial orbit radius

$$R = \frac{\sin(\vartheta + l)}{\sin \vartheta},$$

then we will find that

$$J = \mu L \int_0^{\pi-l} \frac{N f(\vartheta) d\vartheta}{\sin \vartheta}.$$

Since

$$f(\vartheta) = 1 + \cos^2 \vartheta$$

and in accordance with the density change in the medium of the zodiacal light

/126

$$N = \frac{N_0}{r},$$

we find the following for the ecliptic plane

$$J = L \mu \frac{N_0}{\sin^2 l} \left(\frac{4}{3} + \cos l + \frac{\cos^3 l}{3} \right).$$

The brightness of the sky, expressed in the number of stars of the fifth magnitude per square degree, is

$$j = J \left(\frac{\pi}{180} \right)^2.$$

The brightness of the sun in the same units, observed from the earth, is

$$j_{\odot} = 2.512^{31.5}.$$

Consequently,

$$j = \frac{\mu}{\sin^2 l} 2.512^{31.5} \left(\frac{\pi}{180} \right)^2 \left(\frac{4}{3} + \cos l + \cos^3 l \right) N_0,$$

while

$$\mu = \frac{e^4}{2 m^2 c^4} = 0.4 \cdot 10^{-25}.$$

Thus

$$j = 0.18 \cdot 10^{2.4} \frac{N_0}{\sin^2 l} \left(\frac{4}{3} + \cos l + \cos^3 l \right).$$

For $l = 90^\circ$ we have approximately $j = 1$. Thus

$$N_0 = 4 \cdot 10^{2.4} \approx 1000,$$

in accordance with the statements given above. Thus, the actual number of free electrons in interplanetary space per cubic centimeter must be comparatively small, in all probability on the order of not more than several tens. This means that all the gases must occur in an ionized state.

If it is assumed that the output of meteoric matter and its conversion into a gaseous state close to the sun is approximately 10^{18} grams each one hundred thousand years, then the amount of free electrons would have to be quite significant throughout the lifetime of the solar system. In actuality, the liberated electrons remain in the vicinity of the sun and increase the brightness of the outer corona. In order that this increase remain within reason, it is necessary that in the previous epochs during the lifetime of the solar system the density of matter in the zodiacal light would have to be of the same order of magnitude as it is at the present time.

CHAPTER 8

Density Determination in the Medium of Interplanetary Meteoric Matter on the Basis of Stationary Conditions

The motion of an individual meteoric dust particle under the influence of solar attraction and light repulsion was investigated above. It was shown that the dimensions of the orbit gradually decrease, as a result of which each dust particle finally falls into the sun. However, it is quite probable that the total complex of meteoric matter remains in a stationary state, being continually replenished by one or another mechanism. Independently of the manner in which this replenishment by meteoric matter takes place, it can be shown that the stationary condition alone can determine the density distribution at different distances from the sun.

/128

Let us first assume that meteoric particles move along circular orbits. It was shown above that these orbits retain their circular form, that their dimensions will gradually decrease. Let us postulate the condition that the same amount of matter is always found per unit of volume in space at the distance from the sun r .

Let us use $\frac{dr}{dt}$ to designate the rate at which the radius vector of the circular orbit decreases. Apparently, the volume $\omega r^2 \frac{dr}{dt}$ of matter approaching the sun penetrates per unit of time. If we use $f(r)$ to designate the density of matter in space, the amount of matter in this volume will be

/129

$$\omega r^2 \frac{dr}{dt} f(r).$$

The stationary condition has the form

$$\omega r^2 \frac{dr}{dt} f(r) = \text{const.}$$

Since

$$\frac{dr}{dt} = A r^{-1} \rho^{-1},$$

where A is a certain constant, the density function $f(r)$ can be expressed as

$$f(r) = \frac{C}{r}.$$

Thus, when particles perform circular motion, the density of

matter in the over-all dust complex, which exists under stationary conditions, must change as r^{-1} .

Let us examine the more general case of elliptical motion. Let the initial values of the elliptical elements be a_0, e_0 , while at the moment t we have

$$\frac{a}{a_0} = \left(\frac{e}{e_0} \right)^{1/2} \frac{1 - e_0^2}{1 - e^2}.$$

Thus, at each given moment the magnitude of the eccentricity can be regarded as the function of the semimajor axis and of the initial values for the elements. On the other hand,

$$\frac{da}{dt} = -\frac{2B}{mca} \left(1 + \frac{e^2}{2} \right) (1 - e^2)^{-3/2},$$

where

$$B = \frac{\pi \rho^2}{c} r_0^2 E_0$$

with the notation presented above.

Let us investigate a volume of space at the distance r from the sun. All orbits which satisfy the condition

/130

$$a(1 - e) \leq r \leq a(1 + e)$$

can penetrate this volume.

Consequently, the group of orbits (a, e) , which is uniformly distributed around the sun, forms the field $a(1 - e) - a(1 + e)$, which includes the value of the radius vector r .

The density of matter in this zone is proportional to the following factors (see the chapter "Integral Effect of the Decay of Periodic Comets"):

$$\frac{\sin \alpha}{r \sin 2\alpha}, \quad 1)$$

where α is the angle between the radius vector and the tangent to the orbit at a given point,

$$\frac{r \sin \alpha}{\sqrt{a(1 - e^2)}}, \quad 2)$$

$$\frac{1}{r}. \quad 3)$$

The first factor characterizes the quantity which is the inverse of the distance between contiguous orbits; the second - the density

of particles at a given section of the orbit; the third - the degree to which the orbital planes converge.

Thus, the spatial density D , determined by the group of orbits (a, e) , is proportional to

$$D = k \frac{\operatorname{tg} \alpha}{r \sqrt{a(1-e^2)}}.$$

If the condition is introduced that the amount of matter distributed along the orbit does not depend on its dimensions, we then find that

$$k = \frac{C}{a^{3/2}}.$$

Thus, we have

/131

$$D = C \frac{\operatorname{tg} \alpha}{r a^{3/2} \sqrt{a(1-e^2)}}.$$

We should note that

$$\operatorname{tg} \alpha = \sqrt{\frac{ap}{2ar - r^2 - ap}},$$

where p is the parameter of the orbit.

It was indicated above that the total density in a given volume is determined by all the orbits, beginning with those for which

$$a_1(1+e_1) = r,$$

up to those for which

$$a_2(1-e_2) = r.$$

Let $P_a da$ be the probability for the distribution of the orbits ranging from a to $a + da$. The desired density of matter will be

$$\Delta = \frac{1}{r} \int_{a_1}^{a_2} \frac{P_a da \sqrt{ap}}{a^{3/2} \sqrt{p} \sqrt{2ar - r^2 - ap}}$$

or

$$\Delta = \frac{1}{r} \int_{a_1}^{a_2} \frac{P_a da}{a} (2ar - r^2 - ap)^{-1/2}.$$

Let us now introduce the stationary condition. If the group

(a, e) is continuously formed from the group (a_0, e_0) , then under stationary conditions at each interval da the number of orbits must be inversely proportional to $\frac{da}{dt}$.

Consequently, we have

$$\Delta = \frac{1}{r} \int_{a_1}^{a_2} \frac{da}{a \frac{da}{dt} \sqrt{2ar - r^2 - ap}}.$$

It can be readily established that in the case of a circular orbit this expression is simply proportional to r^{-1} , as we pointed out above. Let us now assume that e is a small quantity, so that e^2 can be disregarded in comparison with 1.

/132

In this case

$$\frac{e}{e_0} = \left(\frac{a}{a_0} \right)^{3/4}.$$

In addition,

$$a \frac{da}{dt} = \text{const.}$$

Taking the fact into account that

$$r = a(1 - e^2)(1 - e \cos v)^{-1} = a - ae \cos v,$$

we find

$$\sqrt{2ar - r^2 - ap} = ae \sin v$$

and

$$da = re \sin v dv \left(1 + \frac{3}{4} \frac{r}{a} e \cos v \right).$$

Therefore, we have

$$\Delta = \frac{1}{r} \int_0^\pi \frac{re \sin v dv}{ae \sin v} \left(1 + \frac{r}{a} \cdot \frac{3}{4} e \cos v \right)$$

or after integration

$$\Delta = \int_0^\pi \frac{dv}{a} + \frac{3}{4} \int_0^\pi \frac{r}{a^2} e \cos v dv = \frac{C_1}{a}.$$

Let us now turn to the general expression

$$\Delta = \frac{1}{r} \int_{a_1}^{a_2} \frac{da}{a \frac{da}{dt} \sqrt{2ar - r^2 - ap}}$$

and let us try to retain small quantities of the second order with

/133

respect to e^2 .

We have

$$a \frac{da}{dt} = -K \left(1 + \frac{e^2}{2}\right) (1 - e^2)^{-1/2}$$

or, expanding in series,

$$a \frac{da}{dt} = -K(1 + 2e^2).$$

In a similar way, we obtain

$$\sqrt{2ar - r^2 - ap} = ae \sin v \left(1 - e \cos v - \frac{1}{2}e^2 + e^2 \cos^2 v\right)$$

and

$$re \sin v dv = -da \left[1 - e^2 - (2ae + r \cos v) \frac{3}{4} \frac{e}{a}\right].$$

The expression for Δ can be transformed into the form

$$\begin{aligned} \Delta &= \frac{1}{r} \int_0^\pi \frac{r}{a} \frac{dv}{\left[1 - e^2 - (2ae + r \cos v) \frac{3}{4} \frac{e}{a}\right]} \rightarrow \\ &\rightarrow \frac{dv}{\times (1 + 2e^2) \left(1 - e \cos v - \frac{1}{2}e^2 + e^2 \cos^2 v\right)} \end{aligned}$$

or, after the necessary transformations,

$$\Delta \sim \frac{1}{r} \int_0^\pi dv \left(1 + \frac{3}{4}e \cos v - \frac{3}{16}e^2 \cos^2 v\right).$$

After integration we obtain:

$$\Delta = \frac{C}{r} \left(1 - \frac{3}{32}e^2\right).$$

Since the mean eccentricity $\bar{e}$ decreases with a decrease in the orbital dimensions, the desired density Δ increases somewhat more rapidly than r^{-1} .

/134

Utilizing the approximate relationship

$$e = e_0 \left(\frac{a}{a_0}\right)^{1/4},$$

we finally find

$$\Delta = \frac{C}{r} \left[1 - \frac{3}{32}e_0^2 \left(\frac{a}{a_0}\right)^{1/2}\right] = \frac{C}{r} \left[1 - \frac{3}{32}e_0^2 \frac{r^{1/2}}{a_0^{1/2}}\right].$$

It can be assumed that a_0 corresponds to the distance of the asteroid ring, i.e., it approximately equals 3 - 4 astronomical units.

When compared with the observed density distribution in the medium of zodiacal light, the value of r must usually be taken as less than unity. The mean value is $\frac{r}{a_0} \sim 0.2$.

The formula presented above shows that the additional term which it contains does not exceed 1% for all values of e_0 up to 0.4.

Thus, in this case it can be assumed with sufficient accuracy that

$$\Delta \sim r^1$$

If the brightness actually changes more sharply, then this can serve as an indication that there is a significant amount of orbits with large eccentricities.

CHAPTER 9

Theoretical Determination of Spatial Density of Meteoric Matter in Interplanetary Space

The different mechanisms leading to the accumulation of meteoric /135
matter in interplanetary space were studied in the previous section.
It was found that in every case without exception - whether it were
disintegration of periodic comets captured by planets as a result of
large perturbations, liberation of cosmic dust from the surface of
asteroids at a rate which permitted accumulation in the vicinity of
the sun, or, finally, meteors falling into the sun as a result of
radiative deceleration - the density in the plane of the ecliptic
changed in a manner which was inversely proportional to distance in
the region which could be observed. It is quite probable that a similar
conclusion would be reached by studying the action of other possible
mechanisms which could lead to interplanetary space being occupied by
matter from the outside. However, it is necessary to postulate a cer-
tain hypothesis as to the origin of meteoric matter in order to deter-
mine the density at any point in space outside of the ecliptic.

The integral effect of the decay of periodic comets cannot be
large, since large perturbations are only slightly probable and can
clearly not provide an adequate output of meteoric matter. In addition,
as was shown above, during such a mechanism this matter would have to be
concentrated close to the plane of the ecliptic, and more probably
close to the general plane of planetary motion.

The continuous accumulation in space of dust which has been /136
liberated from the surface of asteroids is incomparably effective.
The presence of radiative deceleration makes it possible to start with
the assumption of considerable ejection velocities.

No matter how small the relative velocity at which dust is ejected
from the surface of an asteroid, even when they are rather small the
dust particles rapidly fill up the inner regions of the solar system,
while under stationary conditions their density must change according
to the law r^{-1} in any plane.

The dispersion of meteoric dust on both sides of the ecliptic will,
apparently, depend on the distribution of asteroids with respect to the
angle of inclination.

Let us try to determine the function of meteoric dust density in
the form

$$f(\varphi),$$

where φ is the heliocentric latitude of the object under considera-
tion in space. Let us derive $f(\varphi)$ on the basis of the distribution

function of the asteroid orbits with respect to the angle of inclination $F(i)$.

Since radiative deceleration does not change the position of the orbital plane, it can be assumed that meteoric dust, which is liberated at a very small relative velocity, can be characterized by the same distribution $F(i)$.

Let us assume that the observer is located in the center of the sun. The density of matter at the angle φ above the ecliptic is determined by the number of particles included in the interval φ and $\varphi + d\varphi$. The particles moving along an orbit which is inclined toward the ecliptic at the angle i , in such a way that $i \geq \varphi$, are located in this interval during the time dt , which is proportional to the orbit section dl contained between the given limits, for the same period of rotation T ,

$$l = \frac{2\pi}{T} t; \quad dl \sim dt.$$

If we use α to designate the angle between the orbit and the latitudinal circle, we have

$$d\varphi = dl \cos \alpha.$$

On the other hand,

/137

$$\operatorname{tg} \varphi = \operatorname{tg} l \cos \alpha.$$

Thus

$$\frac{d\varphi}{dt} = \frac{\operatorname{tg} \alpha}{\operatorname{tg} l}.$$

Since the area of the zone $d\varphi$ is $2\pi \cos \varphi d\varphi$, we find that the density of matter distribution is proportional to

$$f(\varphi) \sim \frac{F(i) dl dl}{2\pi \cos \varphi d\varphi}.$$

Taking the fact into account that

$$\sin \varphi = \sin l \sin i$$

and

$$\operatorname{tg} l = \frac{\sin \varphi}{\sqrt{\sin^2 l - \sin^2 \varphi}},$$

we find

$$f(\varphi) = \frac{F(l) di}{\sqrt{\sin^2 l - \sin^2 \varphi}}.$$

For all possible angles of inclination i , we have

$$f(\varphi) = \int_{\varphi}^{\frac{\pi}{2}} \frac{F(i) di}{\sqrt{\sin^2 i - \sin^2 \varphi}}.$$

In the special case of uniform distribution of the orbits, $f(\varphi)$ does not depend on φ . Actually, the orbital zone must uniformly occupy the surface of the celestial sphere, and therefore

$$F(i) di = C \sin i di.$$

We have

$$f(\varphi) \sim \int_{\varphi}^{\frac{\pi}{2}} \frac{\sin i di}{\sqrt{\sin^2 i - \sin^2 \varphi}}. \quad (17)$$

Assuming that

/138

$$x = \frac{\cos i}{\cos \varphi},$$

in this case we find

$$f(\varphi) = \int_0^1 \frac{dx}{\sqrt{1-x^2}} = \frac{\pi}{2} = \text{const},$$

as was indicated above.

In order to calculate the integral (17), in the general case it is necessary to determine the analytical expression for the distribution function $F(i)$. The number of asteroids, whose orbital inclination is included in different intervals of the angle i according to data from the Kleine Planeten in 1920, is as follows:

i	$0^\circ-8^\circ$	$8^\circ-16^\circ$	$16^\circ-24^\circ$	$24^\circ-32^\circ$	$32^\circ \dots$
N	372	333	83	18	3

The most comprehensive list at the present time, which contains the orbital elements of 1549 asteroids, is given by the Copernicus Institut, Kleine Planeten, 1943.

The calculations which I performed provide the following distribution with respect to inclination:

i	$0^\circ-5^\circ$	$5^\circ-10^\circ$	$10^\circ-15^\circ$	$15^\circ-20^\circ$	$20^\circ-25^\circ$	$25^\circ-30^\circ$	$30^\circ-35^\circ$	$35^\circ-40^\circ$
N	407	515	371	155	68	24	8	1

It is natural to represent this distribution as the product of $\sin i$ by a certain exponential function which rapidly decreases with i .

A representational attempt using expression

/139

$$F(i) = \sin i e^{-ki}$$

did not lead to this result. Thus, the orbital distribution of asteroids with respect to the angle of inclination does not follow the law of random errors. On the other hand, the simpler expression

$$F(i) = \sin i e^{-ki}$$

corresponds to observations very closely. Thus, for example, expressing i in degrees, for $k = 0.20$ we have the following:

i	0°-5°	5°-10°	10°-15°	15°-20°	20°-25°	25°-30°	30°-35°	35°-40°
$F(i)$ calculated	465	514	327	161	75	33	14	—

The data from this table are shown in Figure 6. As can be seen, the agreement reached is satisfactory.

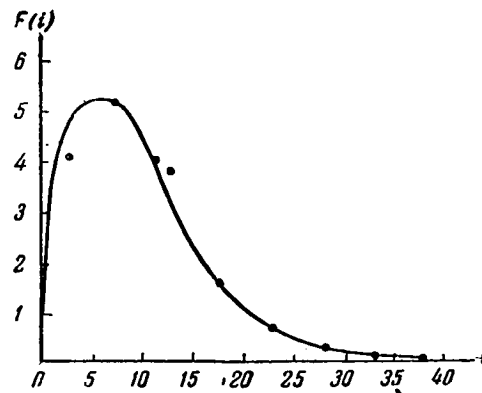


Figure 6.

Distribution of Number of Asteroids with Respect to Their Orbital Angle of Inclination.

Let us now calculate $f(\varphi)$. In the case of small angles of φ , the following approximative computational method can be assumed.

Let us set

/140

$$i = \varphi + \varepsilon.$$

The expression which is proportional to the density

$$f(\varphi) = \int_{\varphi}^{\frac{\pi}{2}} \frac{\sin i e^{-ki} di}{\sqrt{\sin^2 i - \sin^2 \varphi}},$$

can be written in the form

$$f(\varphi) = e^{-k\varphi} \int_0^{\frac{\pi}{2} - \varphi} \frac{\sin(\varphi + \varepsilon) e^{-k\varepsilon} d\varepsilon}{\sqrt{\sin^2(\varphi + \varepsilon) - \sin^2 \varphi}}$$

or, taking the fact into account that

$$\sin^2 i - \sin^2 \varphi = \sin(i - \varphi) \sin(i + \varphi) = \sin \varepsilon \sin(2\varphi + \varepsilon)$$

and assuming that

$$k\varepsilon = x,$$

we find

$$f(\varphi) = \frac{e^{-k\varphi}}{k} \int_0^{k(\frac{\pi}{2} - \varphi)} \frac{\sin\left(\varphi + \frac{x}{k}\right) e^{-x} dx}{\sqrt{\sin \varepsilon \sin(2\varphi + \varepsilon)}}$$

If φ is expressed in radians, as is necessary, then

$$k = 0.20 \cdot \frac{180}{\pi} = 11.5$$

is a very large quantity. Due to this fact, the integral is determined in essence by only small values of the angles ε .

For example, the angle ε , which corresponds to

$$e^{-x} = 0.01,$$

equals

$$\varepsilon = \frac{4.6}{0.2} = 23^\circ.$$

The angles ε , which exceed this value, do not in fact have any effect on the magnitude of the integral.

/141

If $e^{-x} = 0.1$, then $x = 2.3$ and $\varepsilon = 11.5^\circ$.

Due to this fact, all sines under the integral sign can be replaced by the angles for small values of φ . We have approximately in this case

$$f(\varphi) = \frac{e^{-k\varphi}}{k} \int_0^{k(\frac{\pi}{2} - \varphi)} \frac{(\varphi + \varepsilon) e^{-x} dx}{\sqrt{(2\varphi + \varepsilon)\varepsilon}} =$$

$$= \frac{e^{-k\varphi}}{k} \int_0^{k(\frac{\pi}{2} - \varphi)} \sqrt{1 + \frac{\varphi^2}{2\varphi\varepsilon + \varepsilon^2}} e^{-x} dx,$$

from which it follows that

$$f(\varphi) > \frac{e^{-k\varphi}}{k}$$

Since

$$\frac{\varphi^2}{2\varphi\varepsilon + \varepsilon^2}$$

represents a small quantity even for $\varepsilon \geq 3\varphi$, it is possible to simplify the integral to a greater extent and to represent it in the following form

$$f(\varphi) = \frac{e^{-k\varphi}}{k} \left[\int_0^{3k\varphi} \sqrt{1 + \frac{\varphi^2}{2\varphi\varepsilon + \varepsilon^2}} e^{-x} dx + \right.$$

$$\left. + \int_{3k\varphi}^{k(\frac{\pi}{2} - \varphi)} e^{-x} dx \right].$$

In order to avoid the pole of the integrand, we can write

/142

$$f(\varphi) = \frac{e^{-k\varphi}}{k} \left[\int_0^{\eta} \sqrt{1 + \frac{\varphi^2}{2\varphi\varepsilon + \varepsilon^2}} e^{-x} dx + \right.$$

$$\left. + \int_{\eta}^{3k\varphi} \sqrt{1 + \frac{\varphi^2}{2\varphi\varepsilon + \varepsilon^2}} e^{-x} dx + e^{-3k\varphi} \right].$$

The value of η can be determined so that

$$\frac{k^2 \varphi^2}{2k\eta + \eta^2} \gg 1$$

or so that

$$\eta \ll \frac{k\varphi}{2}.$$

If, for example,

$$\eta = \frac{1}{20} k\varphi,$$

then, taking the fact into account that the first integral equals

$$\sqrt{2 k\varphi\eta},$$

we obtain the following approximate expression, which is convenient for numerical calculations

$$f(\varphi) = \frac{e^{-k\varphi}}{k} \left[\frac{k\varphi}{\sqrt{10}} + e^{-3k\varphi} + \int_{\frac{k\varphi}{20}}^{\frac{3k\varphi}{20}} \sqrt{1 + \frac{k^2 \varphi^2}{2 k\varphi x + x^2}} \cdot e^{-x} dx \right].$$

We should note that for $\varphi = 0$

$$f(0) = \frac{1}{k}.$$

For greater values of φ , $f(\varphi)$ can be calculated according to the /143 initial formula

$$f(\varphi) = f(0) e^{-k\varphi} \int_0^{k(\frac{\pi}{2} - \varphi)} \frac{\sin\left(\varphi + \frac{x}{k}\right) e^{-x} dx}{\sqrt{\sin \frac{x}{k} \sin\left(2\varphi + \frac{x}{k}\right)}}.$$

Dividing this integral into two parts

$$\int_0^{k(\frac{\pi}{2} - \varphi)} = \int_0^{x_1} + \int_{x_1}^{k(\frac{\pi}{2} - \varphi)}$$

and assuming that

$$x_1 = k\varepsilon$$

is small as compared with $k\varphi$, we shall have approximately

$$\begin{aligned} f(\varphi) = & f(0) e^{-k\varphi} \int_0^{x_1} \frac{\sin \varphi \sqrt{k} dx}{\sqrt{\sin 2\varphi} \sqrt{x}} + \\ & + \int_{x_1}^{k(\frac{\pi}{2} - \varphi)} \frac{\sin\left(\varphi + \frac{x}{k}\right) e^{-x} dx}{\sqrt{\sin\left(2\varphi + \frac{x}{k}\right)} \sqrt{\sin \frac{x}{k}}} \end{aligned}$$

or

$$f(\varphi) = f(0) e^{-k\varphi} \left[\frac{\sin \varphi \sqrt{kx_1}}{\sqrt{\sin 2\varphi}} + \int_{x_1}^{k(\frac{\pi}{2} - \varphi)} \frac{\sin \left(\varphi + \frac{x}{k} \right) e^{-x} dx}{\sqrt{\sin \left(2\varphi + \frac{x}{k} \right)} \sqrt{\sin \frac{x}{k}}} \right]$$

If, for example, we set

$$x = 0.1,$$

then the corresponding angle is $0^\circ.5$ in all.

Calculations based on this formula lead to the following results: the function $f(\varphi)$ can be represented by the simple expression

$$f(\varphi) = f(0) e^{-k\varphi} R,$$

where $k = 0.20$, R and $f(\varphi)$ are given below:

φ	0°	4°	8°	12°	16°	20°	24°
$f(\varphi)$	1.000	0.544	0.294	0.154	0.081	0.041	0.020
R	1.000	1.211	1.455	1.689	1.987	2.25	2.52

These numbers are represented graphically in Figure 7.

As can be seen, R is a quantity which increases monotonically with φ .

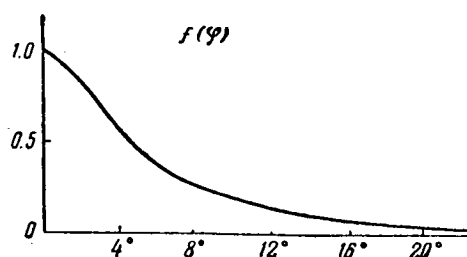


Figure 7.
Function $f(\varphi)$.

According to several considerations, $f(\varphi)$ must be represented

in the form

$$f(\varphi) = f(0) e^{-k_1 \sin \varphi},$$

where k_1 is a constant quantity which can be found empirically.

Let us assume that for all possible φ we have

/145

$$e^{-k_1 \sin \varphi} = K e^{-\kappa \varphi}.$$

We find that

$$k_1 = \frac{k\varphi}{\sin \varphi} = \frac{\lg K}{\lg e} \frac{1}{\sin \varphi}.$$

The values of k_1 are actually sufficiently constant, as can be seen from the following:

φ	4°	8°	12°	16°	20°
k_1	8.7	8.8	9.0	9.1	9.3

Let us assume that $k_1 = 9$ on the average.

The new expression for $f(\varphi)$ represents the calculated values as follows:

φ	0°	4°	8°	12°	16°	20°
$f(\varphi)$	1.00	0.544	0.294	0.154	0.081	0.048
$e^{-9 \sin \varphi}$	1.00	0.533	0.287	0.154	0.084	0.046

Thus, we can finally assume that the density function $D(r, \varphi)$ of meteoric matter which is formed from an asteroidal ring at a negligible initial velocity has the form:

$$D(r, \varphi) = D_0 \frac{e^{-9 \sin \varphi}}{r}.$$

This density distribution is similar to the structure of the atmosphere in hydrostatic equilibrium.

Let us use r, z to designate the distance of the given volume from the sun and the extent to which it surpasses the ecliptic plane, so that

$$z = r \sin \varphi.$$

For given r , the density of matter changes according to the law

/146

$$e^{-z/z_0},$$

where $z_0 = \frac{r}{9}$ plays the role of the altitude of a uniform atmosphere. At the distance from the earth to the sun, this altitude is approximately 17 million kilometers.

It should be noted that the parameter k_1 , which characterizes the hypothetical density distribution in a medium of meteoric matter, is derived under the assumption that the list of asteroidal orbits which are known up to the present encompasses all existing asteroids up to the very smallest ones. The latter are of predominant importance, due to their significant total surface.

However, the possibility is not excluded that the smallest asteroid, which cannot be directly observed, can move along orbits which are more inclined toward the plane of the ecliptic. It is also very probable that dust is liberated at a significant initial velocity, and has an arbitrary orientation with respect to the generating asteroid. This must also lead to the dust being distributed in a wider region of space with respect to the ecliptic.

If we know the function which represents the density of matter in interplanetary space, we can derive the theoretical isophotes for the brightness of zodiacal light.

Let the sun be located at S; the earth - at E; and the object under consideration - at P in space.

Let us use the notation $PS = r$; $EP = \Delta$; $ES = R$.

Let β be the geocentric latitude of P; φ - its heliocentric latitude. Apparently, we have

$$r \sin \varphi = \Delta \sin \beta.$$

If L is the illumination from the sun at the distance R , then the amount of light scattered by the volume dv , which is located at the point P, per unit of solid angle toward the observer will be

/147

$$\mu L \frac{R^2}{r^2} dv D,$$

where μ is the scattering indicatrix.

It thus follows that the brightness of the volume dv observed at the angle $d\omega$, will be

$$\mu L \frac{R^2}{r^2} \frac{dv}{\Delta^2 d\omega} D.$$

Because

$$dv = \Delta^2 d\Delta d\omega,$$

for the total brightness of all dust-like matter in a given direction we have

$$J = LR^2 \int \frac{Dd\Delta}{r^2}.$$

The nature of the scattering indicatrix μ for meteoric matter, which is represented by particles which are large as compared with the length of a light wave, merits a special discussion. Assuming that $\mu = \text{const}$, which must be very nearly the case, and discarding the remaining constant factors, we can represent the observed brightness in the form of the integral

$$J = \int \frac{Dd\Delta}{r^2}.$$

Introducing the expression which was determined

$$D = \frac{e^{-k_1 \sin \varphi}}{r},$$

we find

$$J = \int \frac{e^{-k_1 \sin \varphi} d\Delta}{r^2}$$

or because

/148

$$r^2 = \Delta^2 + R^2 - 2R\Delta \cos \beta \cos (l - l_\odot),$$

where $l, l_\odot$ are the geocentric longitudes of P and S

$$\begin{aligned} J &= \int \frac{e^{-k_1 \frac{\Delta \sin \beta}{r}} d\Delta}{r^2} \\ J &= \int_{R \sin \gamma}^R \frac{e^{-\frac{k_1 \sin \beta}{r}} (R \cos \gamma - \sqrt{r^2 - R^2 \sin^2 \gamma})}{r^2 \sqrt{r^2 - R^2 \sin^2 \gamma}} dr + \\ &+ \int_{R \sin \gamma}^{\infty} \frac{e^{-\frac{k_1 \sin \beta}{r}} (R \cos \gamma + \sqrt{r^2 - R^2 \sin^2 \gamma})}{r^2 \sqrt{r^2 - R^2 \sin^2 \gamma}} dr, \end{aligned} \quad (17)$$

while

$$\cos \gamma = \cos \beta \cos (l - l_{\odot}).$$

The angle γ represents the angular distance of the element P from the sun.

If $\beta = 0$, and consequently $\gamma = l - l_{\odot}$, we have

$$J = \int_{R \sin \gamma}^R \frac{dr}{r^3 \sqrt{1 - \frac{R^2}{r^2} \sin^2 \gamma}} + \int_R^{\infty} \frac{dr}{r^3 \sqrt{1 - \frac{R^2}{r^2} \sin^2 \gamma}}$$

from which it follows that, after integration, we have

$$J = \frac{1}{1 - \cos \gamma},$$

as was found above.

In order to calculate the general integral (17) by a numerical method, we can start with the initial expression

$$J = \int_0^{\infty} \frac{e^{-h_i \frac{\Delta}{r} \sin \beta}}{r^4} d\Delta,$$

while

/149

$$r^2 = 1 + \Delta^2 - 2 \Delta \cos \beta \cos (l - l_{\odot}).$$

It is assumed that the distance between the earth and the sun is unity. Calculations based on the trapezoid method, which were carried out over the interval Δ which equals 0.2, yielded the following results:

$$k_1 = 9.$$

TABLE 15

$\sin \beta$	β	γ	$l - l_{\odot}$	J
0.00	0°	60°	60°	1.97
—	20°	60	7.3	0.362
—	20°	30	23	0.428
0.00	0	60	60	1.97
0.05	2.9	60	60	1.43
0.10	5.8	60	59.8	1.03
0.20	0.5	60	59.3	0.63
0.00	0	20	30	7.45
0.05	2.9	30	29.8	3.94
0.10	5.8	30	29.5	2.24
0.20	11.5	30	28.0	0.94

These numbers make it possible to form an idea of the general nature of isophotes.

These isophotes follow the planes of the ecliptic comparatively closely, but expand in a definite manner as they approach the sun - as is observed in actuality. Thus, the general properties of the phenomenon representing a complex of meteoric dust resulting from the disintegration of asteroids definitely correspond to reality. It should be noted that, on the basis of the same table, it is possible to obtain isophotes which are much more extended, if only the constant k_1 is decreased correspondingly.

Figure 8 illustrates such isophotes, calculated in the same way for $k_1 = \frac{9}{3.4}$. As can be seen, these isophotes closely repre-

/150

sent the actual form of zodiacal light. Thus, the entire complex of meteoric dust, which produces the phenomenon of zodiacal light, is incomparably more widespread in interplanetary space along both sides of the ecliptic plane than would be the case if it were only the result of radiative deceleration.

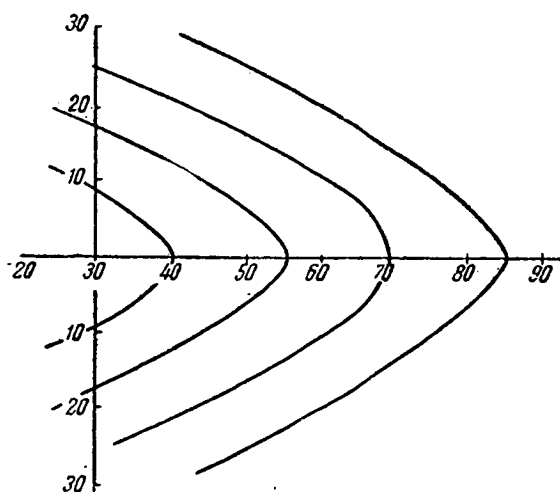


Figure 8.

Theoretical Isophotes of Zodiacal Light.

CHAPTER 10

The Exchange Rate of Zodiacal Light Matter

In the preceding section, it was shown that zodiacal light is a product of the gradual disintegration of asteroids as they collide with meteors. The dust-like matter which is thus obtained as the result of radiative deceleration occupies interplanetary space and, scattering the solar light, it produces observable light phenomena. However, it is necessary to show that the assumed mechanism can produce the requisite output of matter in order that the zodiacal light can remain in a stationary state for an indefinitely long period of time, in spite of dust particles continuously falling into the sun.

/151

Let us focus our primary attention on the problem of the distribution of meteors by size. The only way to formulate an opinion on this subject is to analyze the number of meteors having different apparent magnitudes which are observed when they penetrate the terrestrial atmosphere.

Unfortunately, different determinations lead to greatly diverging results.

There is no doubt that the number of meteors rapidly increases as their brightness decreases, and this increase is approximately the same for all stellar magnitudes.

According to Williams (Ref. 43), the increase in the number of meteors is $k = 2.08 \pm 0.10$ for each stellar magnitude. The most recent work by the same author gives $k = 2.29 \pm 0.16$, which represents observations as follows:

/152

mg	1.0	1.5	2.0	2.5	3.0	3.5	4.0	4.5	5.0	5.5
Observed	1	2	0	6	0	1	10	7	9	20
Calculated	0.6	0.8	1.2	1.7	2.5	3.6	5.2	7.6	10.9	15.7
mg	6.0	6.5	7.0	7.5	8.0	8.5	9.0	9.5		
Observed	21	23	51	71	105	78	22	5		
Calculated	22.7	32.7	47.2	68.0	98.3	142	205	296		

A great deal more information is necessary before a definitive opinion can be formed regarding the increase in the number of meteors. Watson (Ref. 44) studied a series of telescopic observations by Butroyd as well as the observations by Öpik using the naked eye (Ref. 45, 46), and found $k = 4$.

Observations by Hoffmeister using the naked eye (Ref. 47) and a series of telescopic observations by Öpik, which were processed by Watson, showed further that k is less than 4, and probably does not exceed 3.

On the basis of this information, it can be assumed that $k = 2.5$, i.e., the number of meteors increases with each successive magnitude proportionally to a decrease in their brightness, and consequently in their mass. It can thus be approximately assumed that the total mass of all meteors within the limits of each stellar magnitude interval is one and the same. This mass is assumed to equal 11.5 kilograms (Watson). As was shown above (see the Introduction), this law leads to a distribution of meteors by size in the following form

$$f(\rho) = C\rho^{-4}.$$

Because this distribution is characteristic for meteors, it can be assumed that it holds for all of interplanetary space. However, dust-like matter which is liberated from the surface of asteroids has a distribution which is initially different, in which smaller particles predominate. Actually, in the simplest case of circular motion, particles having the radius ρ approach the sun per unit of time along the segment $\frac{dr}{dt}$ under the influence of radiative deceleration.

/153

If $f_0(\rho)d\rho$ is the initial number of particles per unit of volume, then under stable, stationary conditions the spatial density will be proportional to

$$\frac{f_0(\rho) d\rho}{\frac{dr}{dt}} = f(\rho).$$

Because

$$\frac{dr}{dt} \sim \frac{1}{\rho},$$

we have

$$f_0(\rho) \sim f(\rho) \rho^{-1}$$

or

$$f_0(\rho) \sim \rho^{-5}.$$

Let us now determine the mass of zodiacal light falling into

the sun during the established time t . The radial velocity of an individual particle having the radius ρ , at which it approaches the sun, is

$$\frac{dr}{dt} = \frac{A}{r\rho}.$$

Performing integration within the limits from r to 0, we find

$$r^2 = \frac{2A}{\rho} t.$$

It must be noted that during the given time t all the particles having the radius ρ , which are included within a sphere having the radius r , fall into the sun, so that /154

$$r = \sqrt{\frac{2A}{\rho} t}.$$

Let us use $F(\rho)d\rho$ to designate the number of particles per unit of volume in the ecliptic plane and at the distance of the earth from the sun. At any point in space having the coordinates with respect to the sun (r, φ) , we have

$$\frac{F(\rho) d\rho e^{-k_1 \sin \varphi}}{r}.$$

Using δ to designate the particle density, we have the following for the corresponding mass

$$dD = \frac{4}{3} \pi \rho^3 \delta \frac{F(\rho) d\rho e^{-k_1 \sin \varphi}}{r}.$$

Throughout the entire sphere having the radius r , the total mass of matter is

$$\frac{4}{3} \pi \rho^3 \delta F(\rho) d\rho \iint \frac{e^{-k_1 \sin \varphi}}{r} dv,$$

where

$$dv = 2 \pi r^2 \cos \varphi d\varphi dr.$$

Performing integration, we find

$$dM = \frac{4}{3} \pi^2 \rho^3 \delta F(\rho) d\rho \frac{1}{k_1} (1 - e^{-k_1}) r^2.$$

It is interesting to note that in the case of spherical distribution of matter

$$dM_0 = \frac{4}{3} \pi^2 \rho^3 \delta F(\rho) d\rho \cdot r^2$$

so that the ratio between the mass in the actual configuration of zodiacal light and the mass of a spherical configuration is

$$\frac{dM}{dM_0} = \frac{1 - e^{-k_1}}{k_1}.$$

Because

/155

$$k \sim 3 - 2.5;$$

$$\frac{1 - e^{-k_1}}{k_1} \approx 0.3,$$

which does not change the order of magnitude.

The total mass for all sizes of particles, falling into the sun during the time t , is

$$M' = \frac{4}{3} \pi^2 \delta \frac{1}{k_1} (1 - e^{-k_1}) 2 At \int_{\rho_0}^{\infty} \rho^2 F(\rho) d\rho.$$

As was indicated above, assuming that

$$F(\rho) = C\rho^{-4},$$

we find

$$M' = \frac{4}{3} \pi^2 \delta \frac{1}{k_1} (1 - e^{-k_1}) 2 At \frac{C}{\rho_0},$$

where ρ_0 is the lower limit for the particle radius.

On the other hand, the density of matter in zodiacal light, including all sizes of particles from ρ_0 to ρ_1 , is represented by the expression

$$D = \frac{4}{3} \pi \delta C \frac{e^{-k_1 \sin \varphi}}{r} \int_{\rho_0}^{\rho_1} \frac{d\rho}{\rho} = \frac{4}{3} \pi \delta C \frac{e^{-k_1 \sin \varphi}}{r} (\ln \rho_1 - \ln \rho_0).$$

Integrating this expression over the volume of a sphere having the radius r , we find the quantity of mass M_0 contained in it

$$M_0 = \frac{4}{3} \pi^2 \delta r_0^2 C \frac{1}{k_1} (1 - e^{-k_1}) (\ln \rho_1 - \ln \rho_0).$$

Consequently, the portion of the mass falling into the sun during the time t equals

$$\frac{M'}{M_0} = \frac{2 At \rho_0^{-1}}{r_0^2 (\ln \rho_1 - \ln \rho_0)}.$$

In order to determine A, we can find the expression for $\frac{dr}{dt}$.

/156

In the case of circular motion

$$v^2 = \frac{k^2 M'_{\odot}}{r},$$

where $M'_{\odot}$ is the reduced mass of the sun

$$M'_{\odot} = M_{\odot} \left(1 - \frac{3}{4} \frac{r_0^2 E_0}{\rho \delta k^2 M_{\odot}} \right)$$

(k^2 is the gravitation constant; $4\pi r_0^2 E_0$ is the total radiation of the sun per unit of time).

For the values of ρ under consideration, the magnitude of $M'_{\odot}$ barely differs from $M_{\odot}$.

The tangential component of the light pressure impulse per particle at the distance r is

$$\frac{\gamma \pi \rho^2 E_0 r_0^2}{c r^2}$$

and changes the momentum $m \frac{dv}{dt}$, so that

$$m \frac{dv}{dt} = \frac{\gamma \pi \rho^2 E_0 r_0^2}{c r^2},$$

where $\gamma = \frac{v}{c}$ is the aberration constant.

Finding the connection between dv and dr , we finally obtain

$$\frac{dr}{dt} = \frac{3}{2} \frac{E_0 r_0^2}{c^2 \delta r \rho}.$$

It thus follows that

$$A = \frac{3}{2} \frac{E_0 r_0^2}{c^2 \delta}$$

and therefore

$$\frac{M'}{M_0} = \frac{3 E_0 t \rho_0^{-1}}{c^2 \delta (\ln \rho_1 - \ln \rho_0)}.$$

Introducing the following numerical values in this formula:
 $4\pi r_0^2 E_0 = 4 \cdot 10^{33}$ erg/sec; $c = 3 \cdot 10^{10}$ cm/sec; $\rho_1 = 1$ cm; $\rho_0 = 10^{-3}$ cm,
 we find:

$$\frac{M'}{M_0} = \frac{2.1}{\delta} 10^{-5} t,$$

/157

if the time is expressed in years.

Thus, in 100,000 years approximately the entire mass of zodiacal light, included in a sphere having the radius of one astronomical unit, falls into the sun.

This comprises a mass of approximately 10^{18} grams, which is equivalent to an asteroid having a diameter of 10 km. Such an asteroid is fairly representative of this type of small bodies.

Such is the output of zodiacal matter. What can its input be? It can be expected that a large amount of dust-like matter will enter into interplanetary space as a result of asteroids being bombarded by meteors. The number of meteors entering the earth's atmosphere each day was calculated as $2 \cdot 10^7$ at the beginning of the 20th century (Ref. 48). According to Williams, a stream of meteors which is brighter than the 6th magnitude comprises $7.3 \cdot 10^7$ per day for the entire earth, and according to Watson even comprises $1.3 \cdot 10^8$ for the same magnitude interval. This corresponds to meteors whose mass exceeds 0.002 grams.

In one year this amount is $5 \cdot 10^{10}$, and in 10^5 years it is $5 \cdot 10^{15}$ meteors.

It was shown above that the surface of asteroids must exceed the earth's surface by several orders of magnitude. For the same density of meteoric matter in space, the number of impacts on the surface of asteroids comprises $10^{19} - 10^{20}$ on the whole.

Consequently, each impact must yield 0.1 - 0.01 grams of matter, which is fully possible.

Thus, it is possible that each 100,000 years a mass equaling the mass of an asteroid, having a diameter of 10 km, falls into the sun. Throughout the lifetime of the solar system, this would be $10^3 - 10^4$ times greater, i.e., a large portion of the matter in the present-day ring of asteroids.

CHAPTER 11

Zodiacal Light According to Visual Observations and Considerations on Its Photometric Study

The theoretical conclusions presented in the preceding chapters must be compared with observations. There have been an enormous amount of observations on zodiacal light, but in the overwhelming majority of cases, these observations were simply performed by the naked eye. The observations of Heis, Weber (Ref. 49), Jones (Ref. 50), and others which extended over several years are well-known. These observations primarily consisted of sketches of the position of the "outer shape" of this phenomenon and the alignment of its axis with respect to the ecliptic plane. Such observations lead to incorrect concepts regarding the outer form, and even regarding the position of zodiacal light. At our latitudes, zodiacal light appears in the form of cones - evening or morning - which are greatly inclined toward the horizon. Due to the fact that its lower portions are attenuated to a greater extent by atmospheric absorption, the lines of maximum intensity are shifted upwards and close to the horizon by several degrees. As a result, a purely fictitious inclination of the zodiacal light toward the ecliptic is produced (Ref. 51), and in addition the entire phenomenon is displaced toward the north of the ecliptic in the Northern Hemisphere - and toward the south, in the Southern Hemisphere. Because these characteristics are intensified as the inclination of the zodiacal light toward the horizon increases, and because this inclination increases with latitude, it is usually stated that the zodiacal light moves increasingly away from the ecliptic in the same direction in which the observer moves. Some researchers (Ref. 52) have even concluded that the displacement of the zodiacal light from the ecliptic comprises approximately $\frac{1}{15} - \frac{1}{20}$ the latitudinal displacement of the observer. I developed a method (Ref. 53) for determining the effect of atmospheric absorption on the position of the zodiacal light axis, and used it to evaluate the numerous observations of Heis and Weber from 1847 to 1875. The corresponding correction was quite large. The magnitude of the correction, which changed with the time of year, could amount to 4° close to the horizon for particularly unfavorable circumstances, and rapidly decreased with altitude..

Plotting all of the observed positions of the zodiacal light axis on a graph and comparing them for the effect of atmospheric absorption, I found the following elements using the method of least squares: inclination toward the ecliptic, 0.2° ; distance

from the ecliptic to the north 1.0° . Thus, according to the visual observations of Heis and Weber, which were performed in Aachen and Münster and which were corrected by my method, the zodiacal light is located parallel to the ecliptic at a distance of 1° to the north. In addition, an analysis of the same observations showed that the elements of the zodiacal light axis change somewhat with time. It can be concluded from the individual series of observations performed by Heis that the axis of the zodiacal light is sometimes located to the south of the ecliptic. Its inclination fluctuates within narrow limits, but on the average this fluctuation is very negligible. If the indicated correction is not taken into account, then the inclination of the zodiacal light toward the ecliptic is quite significant. Thus, for example, the same observations of Heis on evening zodiacal light yielded an inclination of about 7° . Marchand (Ref. 54) obtained the same inclination of $6 - 7^\circ$ from his observations at the Pic du Midi. Bayldon (Ref. 55) found $i = 4^\circ$; Backhouse (Ref. 56) found $i = 3^\circ$.

At the present time, purely visual observations are performed more or less systematically by numerous astronomy lovers. Since 1937 an observatory has been in operation in Japan, at Ceto (Ref. 57), which is specially designed for such observations. Elvey (Ref. 58) attempted to process material from this observatory, introducing a correction for a change in the ionosphere brightness with zenith distance, and came to the conclusion that zodiacal light changes its brightness regularly within very large limits, depending on the time of year. In general, it was found that in those months when zodiacal light occurs under the best visibility conditions - i.e., when the ecliptic is more steeply inclined toward the horizon - its brightness is greater. According to Elvey, under poorer visibility conditions the brightness of zodiacal light decreases. Elvey found a possible connection between this phenomenon and certain comets. However, there is no doubt that visual observations provide material which is too indefinite and is unsuitable for an accurate discussion.

/160

In order to formulate an opinion regarding the true phenomenon of zodiacal light, independent of airglow and free of the total stellar brightness in our galaxy, it is necessary to perform systematic observations using photometers designed for this purpose. It thus becomes immediately apparent what an important role the terrestrial atmosphere plays in such weak phenomena as zodiacal light, which is not confined to any definite shape. When observing any region in the sky, where the zodiacal light is located, we are actually measuring a group of three different components:

1) the earth's airglow, which is produced in ionosphere layers and, to a certain extent, is also scattered in the troposphere;

2) The total brightness of all the stars comprising our stellar system, which is also absorbed to a certain extent and scattered, particularly in the troposphere;

3) The airglow of the zodiacal light itself, which - as can be seen by its name - is related to the zodiacal zone, i.e., to the ecliptic, and is caused by the dust medium in the solar system.

/161

In order to separate the latter component, in which we are primarily interested, it is necessary to take the first two components into careful consideration. In addition, it is necessary to take into account the absorption and scattering of zodiacal light in the terrestrial atmosphere, and to perform the corresponding reductions. Only after this process, can we expect to obtain material which is comparable with theory. When formulated in this way, the problem proves to be unusually complex, and will not yield to a rigorous solution at the present level of scientific development.

At a later point, I shall attempt to determine the actual brightness and actual system of isophotes for zodiacal light, applying approximative methods which can, however, be overloaded with systematic errors. The difficulty of the problem is aggravated by the fact that the brightness of the zodiacal light represents only a small portion of the observed brightness in regions which are far removed from the sun. The brightness of zodiacal light exceeds the other components of night airglow only in the regions close to the sun. However, these regions close to the horizon can only be observed for a short period of time, are almost impossible to photograph, and are distorted to the greatest extent by atmospheric influences.

First of all, let us try to determine the main component of night airglow which depends on ionosphere emission. In order to do this, it is necessary to know the law adhered to by this airglow as it changes with zenith distance. The height of the emissive ionosphere layer above the earth's surface serves as a basic parameter determining this change. There have been no reliable determinations of this parameter up to the present time. As will be seen from the following information, according to our determinations the height of the emissive layer is about 260-270 km, and almost coincides with the ionization layer F, which always exists under nocturnal conditions, in contrast to other ionization layers. It can apparently be assumed that this height is constant, without a significant error in deriving the brightness of the terrestrial component. The transmission factor of the terrestrial atmosphere serves as another parameter, which changes from night to night and

/162

which must be determined in passing by means of photometric observations on zodiacal light. As of the present, there are no advantageous methods for determining atmospheric transmission, which can change systematically not only throughout the night, but also at different azimuths depending on the properties of the underlying surface. The main obstacle impeding the photometry of zodiacal light is the lack of reliable determinations of atmospheric transmission.

CHAPTER 12

Theory of Night Sky Airglow

In order to draw a distinction between the intensity of the atmospheric component and the over-all night sky airglow, it is necessary to start with the known height of the emissive layer above the surface of the earth. As spectroscopic observations have shown, the night sky airglow is characterized by the bright line 5577A of metastable oxygen, in the green part of the spectrum, the yellow sodium line 5892, and the red oxygen lines 6300 and 6330, which represent two components of a triplet.

/163

The green line dominates in visual rays. In addition, there is a large amount of emissive bands in the more refractile portion of the spectrum; these bands belong to the first positive and second negative series of molecular nitrogen. These bands are apparent only for significant exposure, and can be barely distinguished on the continuous background of the night sky airglow.

Two methods can be used for determining the height of the emissive layer which belongs, apparently, to the ionosphere.

The first method consists of photographing the spectrum of the night sky using a high-transmission spectrograph simultaneously at several zenith distances, beginning from the zenith up to $z = 80^\circ$. By measuring the ratio between the brightnesses of individual emissive lines, one can determine the height of their airglow.

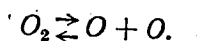
The other method is very similar, and necessitates photometric measurements of the night sky brightness, for example, in common visual rays, also at different zenith distances. However, the assumption is thus introduced that all the primary emissive lines pertain to approximately the same height. In addition, this method requires that the measurements are carried out in absolute units - for example, the measurements are expressed in the number of stars of the fifth magnitude per square degree of sky - since the determinations which are made must not contain the total brightness of stars or other cosmic sources of light.

/164

Generally speaking, a change in the brightness from the zenith to the horizon depends not only on the height of the emissive layer, but also on its thickness. However, the effect of the latter is negligible, according to the studies made by Barbier. If the brightness distribution is not changed as a function of

zenith distance, it can be assumed that the emissive layer has an infinitely small thickness. Elvey derived an accurate formula which took into account the thickness of the layer in several hundreds of kilometers, and he showed that the same result is obtained as in the case of an infinitely thin layer. The derivation of this formula does not entail any difficulties.

Theoretical considerations show that the thickness of the emissive layer must be insignificant. Actually, according to Chepman (Ref. 59), oxygen emissive lines result from the process by which oxygen atoms are dissociated and recombined under the influence of solar radiation. The solar energy dissociates O_2 by day; by night, atoms of O are recombined, and again form O_2 , so that



A third particle N_2 or O thus participates. The energy liberated during the recombination is transmitted to this third particle - for example, O - and excites it to produce the emissive line 5577. This process can take place only in a comparatively thin layer, where there is a good possibility of a simultaneous collision between three particles and where, on the other hand, there is an adequate amount of atomic oxygen.

/165

The brightness of the emissive layer (j) cannot be constant, but changes as a function of φ . We shall therefore assume that

$$j = F(\varphi).$$

Let the line of sight, which is directed from the observer A at the angle of the zenith distance z , intersect the emissive layer at the point B . If the angle between OA and OB at the center of the earth O is α , we then have

$$\sin(z - \alpha) = \frac{\sin z}{1 + h}, \quad (18)$$

while the angle α is a small quantity.

Let us use φ_0 to designate the geographic latitude of the observer; φ - the latitude of a point on the surface of the earth, at the zenith of which the point B is located. Let A be the azimuth of the direction AB .

We have

$$\sin \varphi = \sin \varphi_0 \cos \alpha - \cos \varphi_0 \sin \alpha \cos A$$

or

$$\sin \varphi = \sin \varphi_0 - \alpha \cos \varphi_0 \cos A.$$

We find from (18) that

$$\alpha = \frac{htgz}{1+h} - \frac{h^2}{2} \frac{tg^2 z}{(1+h)^2} + \dots$$

On the other hand,

$$2 \sin \frac{\varphi - \varphi_0}{2} \cos \frac{\varphi + \varphi_0}{2} = -\cos \varphi_0 \cos A \cdot \alpha - \sin \varphi_0 \frac{\alpha^2}{2}.$$

Confining ourselves to the first terms, we have

$$\varphi - \varphi_0 = \Delta\varphi = -\frac{htgz \cos A}{1+h}.$$

It can always be assumed that the brightness of the emissive layer, in the atmospheric region above the level of the observer, changes linearly with latitude. Therefore, it can be assumed that

/166

$$j = F(\varphi_0 + \Delta\varphi) = F(\varphi_0) + \left(\frac{dj}{d\varphi}\right)_0 \Delta\varphi$$

or

$$j = j_0 - \left(\frac{dj}{d\varphi}\right)_0 \frac{h}{1+h} tg z \cos A.$$

If we designate γ as the relative airglow gradient with respect to latitude

$$\gamma = \frac{1}{j_0} \left(\frac{dj}{d\varphi}\right)_0,$$

we find the following expression for the observed airglow of the emissive layer in the direction of the line of sight (z, A), when there is no absorption or scattering in the intermediate layers:

$$J = j_0 \frac{(1+h) \left(1 - \gamma \frac{h}{1+h} \frac{180}{\pi} tg z \cos A\right)}{\sqrt{(1+h)^2 - \sin^2 z}},$$

where j_0 designates the emissive layer brightness at the zenith of the observer. What may be the magnitude of γ ? It is quite probable that the radiance of the night sky increases with latitude. In 1934 I photographed a region in the sky around the celestial pole, where the stellar configuration always remained the same, at Kitabe ($\varphi = 39^\circ 8'$), Tashkent ($41^\circ 20'$), and Kuchino ($55^\circ 45'$) near Moscow

(Ref. 60). A tubular photometer was employed for the photography, which was performed on the same plates with the same exposures but at a different time. The radiance of the sky was much higher at Kuchino than it was at Kitabe and Tashkent. In 1940/41, I made another attempt to determine any change in the radiance of the sky at different latitudes. Systematic observations were organized at Bukhte Tikhoy (N. P. Uspenskiy), at Franz Josef Land, at Kuchino, and at Simenze (P. F. Shayn [Ref. 61]). The observations were performed with multistage radioactive photometers which I constructed. These photometers made it possible to compare the radiance of the sky with self-luminous sections having a different brightness. These sections were made of zinc sulfide with an admixture of a radioactive substance, which served to excite the fluorescence. The photometers, which were produced at the same time, were compared with each other on the optical bench of the Kuchinskiy Astrophysical Observatory. It was found that as the latitude changes from 80° to 45° , the brightness of the sky can be represented linearly. The gradient γ equaled 0.092 for $\varphi = 45^\circ$, which represents a change in the brightness of the sky from 80 to 44° . However, these observations were not absolutely conclusive, since the properties of the underlying surface could not be taken into account. In addition, the Bukhte Tikhoy station was close to the region of maximum frequency of the aurora polaris, which could distort the observational results in many cases. In addition, the illumination of the atmosphere by the sun, which causes the accumulation of energy which is lost during the night, is entirely different in the north than it is in the middle latitudes, particularly during the polar night. In any case, the results derived from these observations - the existence of which cannot be doubted - cannot be directly applied to interpreting the radiance of the night sky at individual points on the earth's surface.

/167

In this connection, we should note that the observations of L. Rayleigh, which were performed at the more southern latitudes, apparently did not evidence any purely latitudinal effect, as can be seen from a discussion on this subject which took place at the Korolevskiy Astronomical Society in 1945 (Ref. 62). It is more reasonable to determine the magnitude of γ by analyzing the radiance at different points of the sky, observed at a given location at one and the same time. We should note that the distance between an observer and a point on the earth's surface, through whose zenith the line of sight passes at the height h for the zenith distance z , is quite large, namely:

$z \backslash h \text{ (km)}$	10	100	300	1000
80°	60	450	1000	2400
85°	110	700	1450	2800

/168

If a comparison is made between the radiance of the sky at different points on the horizon for large zenith distances, it will be possible to discover a distinct latitudinal effect if it exists. However, this type of observations does not lead to definite results. Very frequently the sky is brighter close to the northern horizon than it is at the same altitude close to the southern horizon, but this difference does not usually change with the azimuth according to formula (18) and can depend on different factors. These factors include, for example, the effect of zodiacal light, which can produce additional airglow in the north around midnight, as well as different properties in the underlying surface of the earth. Thus, for example, the brightness of the sky in the northern section of the horizon, in the vicinity of Alma-Ata in October, 1945 was slightly greater than in the south at the same zenith distance. There was clearly no influence from the zodiacal light. The brightness of the sky was almost exactly the same in the west as it was in the south, which contradicts the formula presented above. However, the properties of the underlying surface were completely different in different directions: a dull, sandy desert extended to the north, and a dual range of high mountains which were covered perpetually with snow was located in the south. Nevertheless, the latitudinal effect would have to be quite small. In order to solve the problem, it would be necessary to carry out an adequate number of observations at different azimuths of the sky, close to the horizon and far from the Milky Way and the zodiacal light. The observations would also have to be carried out in a region where the properties of the soil were the same for hundreds of kilometers, preferably in a desert with a low reflecting capacity. For the special purpose of determining the height of an emissive layer, the observations would have to be performed close to the first vertical ($A = 90^\circ$), where there was absolutely no latitudinal effect. The observed brightness of the sky could then be represented in the form

/169

$$I_{90} = \frac{j_0(1+h)(abs)}{\sqrt{(1+h)^2 - \sin^2 z}},$$

where (abs) is the factor expressing absorption and scattering of light in a medium between the ionosphere and the observer.

This factor is primarily determined by low air layers.

In the case of a light source having a small angular dimension, the light absorption is determined by the usual factor $p^{\sec z}$, where p is the transmission coefficient. The position is completely different for extended objects, especially for night sky airglow which occurs at all points in the celestial sphere, since in the given case light which is scattered in the

troposphere is more or less important.

Dufay first focused attention on this in 1928 (Ref. 63), employing a method for determining the diffuse light, which, however, was not accurate. In 1938, Dufay, Kabani, and Gozit also employed a rather cumbersome, semi-empirical method for this purpose, in order to determine the height of the emissive layer. In those cases when such a determination was not carried out, rather strange results were obtained, as occurred in the studies of Abadie, Vassy and Madam Vassy (Ref. 64).

In a recent investigation of the emissive layer height, Elvey (Ref. 65) employed a purely empirical method to determine the diffuse light, not ascribing to it any theoretical basis. Determining the normal transmission coefficient $p = e^{-k}$ from the observations, he reduced the observed brightness of the emissive lines using the factor

$$p_1^{\sec z} = e^{-\frac{k}{2} \sec z}.$$

This procedure by Elvey is only justified by the fact that close /170 agreement is thus obtained between the determinations of the emissive layer height at different points in America, where the study was carried out.

In 1944, D. Barbier (Ref. 66) used a similar method, based on employing a transfer equation, assuming that the atmosphere had purely scattering properties.

If we examine an atmospheric volume $dv = dsdx$ (ds - cross-section of the base, dx is the altitude), through which a stream of radiation J (calculated per unit of solid angle) passes in the x -direction, then the transfer equation - as is known - will be

$$\frac{dI}{dx} = -\sigma I + \sigma J.$$

Here σJ is the amount of radiation scattered by a unit volume in unit of solid angle in the x -direction, out of the total radiation in all directions passing through the given volume.

If it is assumed - as Barbier did - that the scattering proceeds according to the Rayleigh formula, then

$$\frac{16\pi}{3} J = \int (1 + \cos^2 \beta) I d\omega, \quad (19)$$

where β is the angle between the incident and scattered rays. If the radiation is isotropic, then

$$J = I.$$

Let us introduce the optical thickness τ according to the equation

$$d\tau = \sigma dx,$$

where σ is the scattering coefficient.

The transfer equation assumes the form

$$\frac{dI}{d\tau} = -I + J.$$

Assuming that J is constant throughout the entire atmosphere, we have

$$I = J + Ce^{-\tau}.$$

Let us set the following at the boundary of the atmosphere

/171

$$\tau = 0 \quad \text{and} \quad I = I_0.$$

If τ_1 is the optical thickness of the entire atmosphere, we then have

$$I_1 = I_0 e^{-\tau_1} + J(1 - e^{-\tau_1}).$$

The first term represents the observed brightness on the surface of the earth, in the case of a point light source; the second term is the additional brightness representing diffuse light. For

$$\tau_1 \rightarrow \infty, \quad I \rightarrow J.$$

For any point in the atmosphere, we have

$$I = I_0 e^{-\tau} + J(1 - e^{-\tau}).$$

Substituting this in equation (19) and deriving a solution with respect to J , we find

$$J = \frac{\int I_0 e^{-\tau} (1 + \cos^2 \beta) d\omega}{\int e^{-\tau} (1 + \cos^2 \beta) d\omega}.$$

Let (z_1, A_1) be the horizontal coordinates of an observed point in the sky; let (z, A) be the coordinates which determine the direction of a certain stream scattered in a given volume.

It is apparent that

$$\cos \beta = \cos z \cos z_1 + \sin z \sin z_1 \cos(A - A_1).$$

We have

$$I = I_0 e^{-\tau} + \frac{\iint I_0 e^{-\tau} (1 + \cos^2 \beta) \sin \alpha dz dA}{\iint e^{-\tau} (1 + \cos^2 \beta) \sin \alpha dz dA} (1 - e^{-\tau}),$$

where

$$\tau = \tau_0 \sec z,$$

τ_0 is the optical thickness in the vertical direction.

In order to simplify the calculations, Barbier assumes the following under the integral sign

$$\tau = 0.$$

Integration becomes possible, and it is found that

/172

$$\begin{aligned} J_0 &= \frac{\iint I_0 (1 + \cos^2 \beta) d\omega}{\iint (1 + \cos^2 \beta) d\omega} = \\ &= \frac{3}{16} I_0^0 \sqrt{1+a^2} \int_0^1 [3 - \cos^2 z_1 - (1 - 3 \cos^2 z_1) u^2] \frac{du}{\sqrt{a^2 + u^2}}, \\ a^2 &= (1+h)^2 - 1; \quad \cos z = u \end{aligned}$$

(it is assumed that the radius of the earth is unity); I_0^0 - designates the brightness at the boundary of the atmosphere and at the zenith.

It thus follows that

$$J_0 = I_0^0 (F_1 - F_2 \cos^2 z_1),$$

while

$$\begin{aligned} F_1 &= \frac{3}{16} \sqrt{1+a^2} \left[\left(3 + \frac{a^2}{2} \right) \text{sh}^{-1} \frac{1}{a} - \frac{\sqrt{1+a^2}}{2} \right] \\ F_2 &= \frac{3}{16} \sqrt{1+a^2} \left[\left(1 + \frac{3}{2} a^2 \right) \text{sh}^{-1} \frac{1}{a} - \frac{3}{2} \sqrt{1+a^2} \right], \end{aligned}$$

where $\text{sh}^{-1}x$ is the argument whose hyperbolic sine is x .

Incidentally we have

$$\text{sh}^{-1}x = \lg e(x + \sqrt{1+x^2}).$$

Thus,

$$I = I_0 e^{-\tau_0 \sec z_1} + (1 - e^{-\tau_0 \sec z_1}) I_0^0 (F_1 - F_2 \cos^2 z_1).$$

Inserting the value of I_0 , up to the proportionality coefficient

we have

$$I = \frac{(1+h)e^{-\tau_0 \sec z_1}}{\sqrt{(1+h)^2 - \sin^2 z_1}} + (1 - e^{-\tau_0 \sec z_1})(F_1 - F_2 \cos^2 z_1),$$

since $I_0^0 = 1$. According to the calculation of Barbier, the values of F_1, F_2 for different h are:

h (km) . .	50	100	150	200	300	400	500	1000	∞
F_1	1.49	1.30	1.19	1.12	1.03	0.90	0.92	0.78	0.50
F_2	0.25	0.20	0.16	0.14	0.12	0.10	0.09	0.05	0.00

/173

For example, for $h = 300$ km

$$I = \frac{1.047 p^{\sec z_1}}{\sqrt{1.047^2 - \sin^2 z_1}} + (1 - p^{\sec z_1})(1.03 - 0.12 \cos^2 z_1),$$

where p is the normal transmission coefficient.

It can thus be seen that the first term increases, reaches a maximum, and then decreases for large zenith distances, whereas the second term increases monotonically and can have values which are comparable with those of the first term. Under normal conditions (p - on the order of 0.8), the effect of the second term is quite large. The first term yields the first approximation only for an unusually pure atmosphere under mountainous conditions.

It was assumed above that throughout the entire atmosphere $J = \text{const}$. Barbier estimated the approximate change in J throughout the atmosphere, calculating its value at ground level and at the upper boundary, formulating an implicit assumption regarding the monotonic course of J . One simplification is thus made - namely, a hypothesis regarding the spherical form of the scattering indicatrix is thus introduced. In this case, the following is obtained for ground level

$$J_s = \frac{\int_0^{\frac{\pi}{2}} I_0 e^{-\tau_1 \sin z} \sin z dz}{\int_0^{\frac{\pi}{2}} e^{-\tau_1 \sin z} \sin z dz + 1},$$

and at a sufficient height above the emissive layer

$$J_A = \frac{\int_0^{\frac{\pi}{2}} I_0 \sin z dz}{\int_{\frac{\pi}{2}}^{\pi} e^{-\tau_1 \sin z} \sin z dz + 1}.$$

As has been indicated, τ_1 is the optical mass of the atmosphere measured downward; τ_2 is the same mass, but measured upwards. In the case of a spherical indicatrix, $\tau_1 = \tau_2$. The calculations of Barbier lead to the following results: /174

		I_s	I_A	I_0
$h = \infty$	$\tau_0 = 0.011$	0.459	0.541	0.500
	0.043	0.408	0.592	0.500
	0.174	0.272	0.730	0.500
$h = 100 \text{ km}$	0.011	1.06	1.23	1.14
	0.043	0.86	1.95	1.14
	0.174	0.47	1.67	1.14

The mean value of J_0 is determined by the expression

$$J_0 = \frac{1}{2} \int_0^{\frac{\pi}{2}} I_0 \sin z dz.$$

It can thus be seen that under ordinary conditions J changes greatly within the atmospheric limits, and therefore the basic supposition of Barbier is invalid.

Thus, for example, for $p = 0.80$, $\tau_0 = 0.2$. As can be seen from the Table, J changes by several factors between the upper and lower boundaries of the atmosphere independently of the emissive layer height. If it is assumed, as Barbier did, that J changes linearly with height (the second assumption) - namely, if it is assumed that

$$I = I_A - \frac{I_A - I_s}{\tau_1} \tau,$$

- then we obtain the following formula for the brightness of the sky observed from the surface of the earth

$$I_1 = I_0 e^{-\tau_1} + I_A (1 - e^{-\tau_1}) - \frac{I_A - I_s}{\tau_1} (\tau_1 - 1 + e^{-\tau_1}).$$

This formula differs very little from the preceding one only for very small values of τ_1 .

The method assumed by Barbier for determining diffuse light is correct in principle, but due to numerous simplifications it entails systematic errors, which are impossible to evaluate. /175

The primary simplifications are as follows: 1) the hypothesis regarding J as constant throughout the entire atmosphere; 2) the application of the scattering indicatrix of Rayleigh; 3) a disregard of the factor $e^{-\tau} = p^{\sec z}$ under the integral sign when

calculating J_0 .

There is no doubt that very extensive and cumbersome numerical calculations are required even in the case of pure scattering when making a rigorous application of the transfer equation.

The effect of diffuse light can be determined more directly on the basis of the following considerations (Ref. 67).

Let us regard each element in the emissive layer as an upper source of light with respect to the troposphere. According to existing determinations, the effective height of the emissive line airglow is several hundreds of kilometers, while scattering is confined only to a thin atmospheric layer close to the surface of the earth within a range of several kilometers. Observations show that a distant source of light - for example, the sun - illuminates the vault of heaven due to scattering in the troposphere, according to the formula

$$j = kf(\vartheta) \varphi(z, z_1) \sec z_1,$$

where $f(\vartheta)$ is the function which depends only on the angular distance from the sun and which characterizes the scattering indicatrix; $\varphi(z, z_1)$ is the function which depends on the zenith distance of the sun z , the zenith distance of an observed point in the sky z_1 , and the transmission coefficient p .

In the special case of Rayleigh scattering, we have

$$f(\vartheta) = 1 + \cos^2 \vartheta.$$

On the other hand, as the theory for the brightness of the diurnal sky shows - taking into account light scattering of the first order of magnitude - we have

$$\varphi(z, z_1) = \frac{p^{\sec z} - p^{\sec z_1}}{\sec z_1 - \sec z}.$$

The validity of the primary assumption - to the effect that the brightness at any point in the diurnal sky is represented as the product of two functions - is confirmed by observations within an accuracy of several percents. Thus, for example, the observations of Ye. V. Fesenkova - Pyaskovskiy in the vicinity of Alma-Ata in August, 1946 (Ref. 68) yielded the following values for the brightness of the sky at different angular distances from the sun:

/176

TABLE 16

$\begin{array}{c} \vartheta \\ \backslash \\ z \end{array}$	30°	60°	80°	40°	60°	80°
40°	2.10	1.54	1.27	1.49	1.54	1.54
60°	2.91	2.06	1.70	2.06	2.06	2.06
80°	5.01	3.48	3.02	3.55	3.48	3.66

The right half of the table gives the reduced values of the brightness (in every case, for $z = 60^\circ$ and $\vartheta = 60^\circ$ the brightness is assumed to equal 2.06). As can be seen, the brightness distribution at the same angular distance from the sun is the same, and depends only on the zenith distance. In the green and red spectral lines, for which scattering of higher orders of magnitude is significantly less than for blue lines, the given law must be maintained even more precisely. The form of the function $\phi(z, z_1)$ indicated above can be experimentally corroborated by the fact that the value for the transmission index p , obtained with respect to the brightness of the diurnal sky, satisfactorily agrees with the values of p , derived on the basis of the usual formula given by Bouguer.

The illumination of the troposphere at a certain point (z, A) , which depends on the entire emissive layer brightness $j(z, A)$ is

$$J = \sec z_1 \iint j(z, A) f(\vartheta) \frac{p^{\sec z} - p^{\sec z_1}}{\sec z_1 - \sec z} \sin z dz dA,$$

while

$$\cos \vartheta = \cos z \cos z_1 + \sin z \sin z_1 \cos(A - A_1).$$

This expression can be readily calculated, if it is assumed that the emissive layer brightness is everywhere the same, and the scattering indicatrix $f(\vartheta)$ has a spherical form. On the basis of these assumptions

$$J = C \sec z_1 \int_0^{\frac{\pi}{2}} \frac{p^{\sec z} - p^{\sec z_1}}{\sec z_1 - \sec z} \sin z dz.$$

Assuming that

$$x = k \sec z; \quad x_1 = k \sec z_1,$$

we have

$$I = C k^2 x_1 \int_k^\infty \frac{e^{-x} - e^{-x_1}}{x_1 - x} \cdot \frac{dx}{x^2}$$

We should note that secants of zenith distances are used in this expression, instead of atmospheric masses.

Since

$$\frac{1}{(x_1 - x)x^2} = \frac{1}{xx_1^2} + \frac{1}{x_1x^2} + \frac{1}{x_1^2(x_1 - x)},$$

we obtain the following by simple reduction

$$\begin{aligned} \int_k^\infty \frac{e^{-x} - e^{-x_1}}{x_1 - x} \cdot \frac{dx}{x^2} = & -\frac{Ei(-k)}{x_1^2} + \frac{e^{-x_1}}{x_1^2} \ln k + \\ & + \frac{1}{x_1^2} e^{-x_1} [Ei(x_1 - k) - e^{-x_1} \ln(x_1 - k)] + \\ & + \frac{1}{x_1} \left[\frac{e^{-k}}{k} + Ei(-k) - \frac{e^{-x_1}}{k} \right], \end{aligned}$$

while

$$Ei(-k) = - \int_k^\infty e^{-x} \frac{dx}{x} \quad \text{and} \quad k = -\ln p.$$

Finally we find

/178

$$\begin{aligned} \frac{I}{C} = & \cos z_1 p^{\sec z_1} (Ei[k(\sec z_1 - 1)] - \ln[k(\sec z_1 - 1)] + \ln k) - \\ & - p^{\sec z_1} + p + Ei(-k)(k - \cos z_1). \end{aligned}$$

On the other hand, let us assume that the brightness distribution of an emissive layer is most pronounced and follows the secant law, i.e.,

$$j(z, A) = \sec z.$$

Then

$$J = C \sec z_1 \int_0^{\frac{\pi}{2}} f(\vartheta) \frac{p^{\sec z} - p^{\sec z_1}}{\sec z_1 - \sec z} \sin z \sec z dz.$$

Assuming that $f(\vartheta) = \text{const}$, by integrating this expression, we find that

$$I = C \sec z_1 \int_1^\infty \frac{p^x - p^{\sec z_1}}{\sec z_1 - x} \cdot \frac{dx}{x}$$

or

$$I = C \sec z_1 k \int_k^\infty \frac{e^{-x} - e^{-x_1}}{x_1 - x} \cdot \frac{dx}{x}.$$

Since

$$\frac{1}{(x_1 - x)x} = \frac{1}{xx_1} + \frac{1}{(x_1 - x)x_1},$$

we have

$$\begin{aligned} I &= C \sec z_1 \left[\frac{-Ei(-x)}{x_1} + \frac{1}{x_1} \int_k^\infty \frac{e^{-x}}{x_1 - x} dx \right] - \\ &- C \sec z_1 e^{-x_1} \left[\frac{1}{x_1} (\ln \infty - \ln k) + \frac{1}{x_1} \int_k^\infty \frac{dx}{x_1 - x} \right] \\ &\int_k^\infty \frac{e^{-x}}{x_1 - x} dx = e^{-x_1} Ei(x_1 - k) \\ &\int_k^\infty \frac{dx}{x_1 - x} = - \int_{x_1 - k}^\infty \frac{dz}{z} = - \ln \infty + \ln(x_1 - k), \end{aligned}$$

/179

since

$$\int_{-a}^\infty \frac{dx}{x} = \int_a^\infty \frac{dx}{x}.$$

Consequently, we finally have

$$\begin{aligned} J &= C \{ -Ei(-k) + p^{\sec z_1} Ei[k(\sec z_1 - 1)] + p^{\sec z_1} \ln k - \\ &- p^{\sec z_1} \ln[k_1(\sec z_1 - 1)] \}. \end{aligned}$$

We should note that infinity ∞ designates a simple number which is rather large as compared with unity. In fact, in terms of this problem this is the maximum value of the atmospheric mass, i.e., a quantity on the order of 50 - 60.

The following must be taken into consideration when calculating these expressions:

$Ei(0) - \ln(0) = 0.5772$ is the Euler constant.

The following Table gives the relative course for the values of J in both cases:

$\sec z_1$	1	2	3	4
$f = \text{const} \dots\dots$	1.00	1.72	2.32	2.76
$f \propto \sec z \dots\dots$	1.00	1.17	1.30	1.40

As can be seen, the effect of tropospheric diffuse light depends to a high degree on the brightness distribution in the primary light source.

As is known, the effect of light scattering by the troposphere increases toward the horizon, while the observed brightness of the ionosphere increases at first, then - after a definite zenith distance - begins to decrease. The relationship between purely ionospheric and tropospheric airglow depends on the condition of the troposphere, and must be derived from observations. We shall show that the corresponding coefficient of proportionality can be found under our simplifying assumptions as the ratio between the illumination of a horizontal surface from the sun and from the vault of heaven. Actually, let L be a stream of radiation from the sun outside the atmosphere per unit of surface in a normal direction.

/180

Per unit of horizontal surface, the stream of radiation will be

$$L \cos z_1 p^{\sec z}.$$

On the other hand, the brightness of the vault of heaven at the point (z, ϑ) is

$$J = L \frac{\mu}{k} f(\vartheta) \frac{p^{\sec z} - p^{\sec z_1}}{\sec z_1 - \sec z} \sec z$$

and therefore, under the assumption that

$$\frac{\mu}{k} f(\vartheta) = K = \text{const},$$

the illumination of a horizontal surface from the entire vault of heaven will be

$$2\pi L K \int_0^{\frac{\pi}{2}} \frac{p^{\sec z_1} - p^{\sec z}}{\sec z - \sec z_1} \sin z dz.$$

The ratio between these quantities S , which can be readily found from observations, is represented by the formula

$$p^{\sec z_1} S = 2\pi \sec z_1 K \int_0^{\frac{\pi}{2}} \frac{p^{\sec z_1} - p^{\sec z}}{\sec z - \sec z_1} \sin z dz.$$

The right part of the equation represents the ratio between the brightness of tropospheric diffuse light and the brightness of

the stratosphere, and determines the factor K.

Let us examine this problem in the general case.

/181

As was shown above, the observed radiance of the ionosphere can be represented by the formula

$$J = \frac{j_0(1+h)p^{\sec z} \left(1 - \gamma \frac{180}{\pi} \frac{h}{1+h} \operatorname{tg} z \cos A\right) (1+X)}{\sqrt{(1+h)^2 - \sin^2 z}}$$

or

$$J = j_0 F(z, A) (1+X) p^{\sec z},$$

where j_0 is the brightness of the ionosphere at the zenith; X is the increase in brightness due to diffuse light in the troposphere.

Let us show that the function $X(z_1, A)$ at the point z_1, A is connected with the function $K(z_1)$, which is found from direct observations by the following simple dependence

$$X(z_1, A) = K(z_1) \frac{\overline{F(z_1)}}{F(z_1, A)},$$

while $K(z_1)$ represents the ratio between the illumination of a horizontal surface from the diurnal sky and the illumination of the same surface from the sun, at the zenith distance z_1 .

Let L be the illumination from the sun outside of the atmosphere in a normal direction. The observed illumination on a horizontal surface will be

$$L \cos z_1 p^{\sec z_1}.$$

The observed illumination from the sky on the same surface is

$$\iint L \frac{\mu}{k} f(\vartheta) \frac{p^{\sec z_1} - p^{\sec z}}{\sec z - \sec z_1} \sec z \cos z \sin z dz dA.$$

Consequently,

$$K = \frac{\mu}{k} \sec z_1 p^{-\sec z_1} \iint f(\vartheta) \frac{p^{\sec z_1} - p^{\sec z}}{\sec z - \sec z_1} \sin z dz dA.$$

We shall regard an element in the ionosphere having the brightness $j_0 F(z, A)$ as an independent source of light outside of the troposphere. The brightness of the troposphere, produced by

/182

the entire ionosphere in the direction (z, A_1) , is

$$H = \frac{\mu}{k} j_0 \iint F(z, A) f(\vartheta) \frac{p^{\sec z_1} - p^{\sec z}}{\sec z - \sec z_1} \sec z_1 \sin z dz dA.$$

If diffuse light is not taken into account, the brightness of the ionosphere is

$$I = j_0 F(z_1, A) p^{\sec z_1}.$$

The ratio between these quantities is

$$X = \frac{H}{I} = \frac{\frac{\mu}{k} \sec z_1}{p^{\sec z_1}} \frac{\iint F(z, A) f(\vartheta) \frac{p^{\sec z_1} - p^{\sec z}}{\sec z - \sec z_1} \sin z dz dA}{F(z_1, A_1)}.$$

It thus follows that

$$\frac{X}{K} = \frac{\iint F(z, A) f(\vartheta) \frac{p^{\sec z_1} - p^{\sec z}}{\sec z - \sec z_1} \sin z dz dA}{F(z_1, A_1) \iint f(\vartheta) \frac{p^{\sec z_1} - p^{\sec z}}{\sec z - \sec z_1} \sin z dz dA} = \frac{F(z_1)}{F(z_1, A_1)}.$$

In the case of uniform ionosphere brightness $F(z, A) = \text{const}$

$$X = K.$$

The magnitude of K is found from direct observations. The

ratio $\frac{F(z_1)}{F(z_1, A_1)}$ must be calculated.

It was indicated above that the latitudinal effect of the night sky radiance must be very small. Therefore, from this point on we shall assume that $F(z, A)$ depends only on the height of the emissive layer, and is represented by the simple formula

$$F(z, A) = \frac{1 + h}{\sqrt{(1 + h)^2 - \sin^2 z}}.$$

Let us now calculate X .

The scattering indicatrix $f(\vartheta)$ depends on the nature of the air mass. In the European portion of the USSR, under the conditions of a comparatively humid climate, when greatly extended ahead, the indicatrix is also extended to the rear. In central Asia, where there is a dry and dusty atmosphere, the indicatrix is extended ahead to a much greater extent - while its extension to the rear

/183

is correspondingly less - which causes good visibility conditions at the side opposite the sun and, on the other hand, very poor visibility toward the sun. However, in rainy seasons the scattering indicatrix in Central Asia approximates a form which is characteristic for the European portion of the USSR, and is even less asymmetric - as was indicated by Ye. V. Fesenkoy-Pyaskovskiy in the region of Talgar at an altitude of 1,400 m for 1946, which was abnormal in terms of precipitation.

Therefore, I decided upon the average form for the scattering indicatrix to be used in the numerical calculations. This was obtained by Ye. V. Fesenkoy-Pyaskovskiy (Ref. 69) in July, 1945, in the vicinity of the city of Ivanovo.

On the basis of these figures, the function $f(\vartheta)$ can be represented by the following empirical formula

$$f(\vartheta) = 1 + 1.652 \cos^2 \vartheta + 1.214 \cos^3 \vartheta,$$

which represents the observations quite well, as can be seen from Table 17.

TABLE 17

ϑ	15°	20°	30°	40°	60°	70°	80°	90°	110°	120°	130°	150°	
$f(\vartheta)$	3.63	3.47	3.03	2.52	1.56	1.24	1.06	1.00	1.14	1.26	1.36	1.45	calc.
$f(\vartheta)$	3.91	3.24	2.48	2.17	1.41	1.25	1.09	1.00	1.03	1.13	1.29	1.45	obs.

Let us set in general

$$f(\vartheta) = \sum_{i=0}^n a_i \cos^i \vartheta,$$

while $a_0 = 1$ according to the normalization condition.

/184

It is necessary to calculate the expression

$$\iint F(z) f(\vartheta) \varphi(z, z_1) \sin z dz dA,$$

while

$$\cos \vartheta = \cos z \cos z_1 + \sin z \sin z_1 \cos A.$$

Let us designate

$\overline{F(z_1)}$.

These values of $\overline{F(z_1)}$ are given in Table 18.

TABLE 18.

$h = 0.04$						
$\begin{matrix} \text{sec } z \\ p \end{matrix}$	1	1.5	2	3	4	6
0.77	1.66	1.80	1.895	1.855	1.855	1.85
0.80	1.68	1.82	1.855	1.875	1.875	1.875
0.83	1.70	1.84	1.875	1.905	1.905	1.90
0.86	1.71	1.86	1.90	1.925	1.93	1.93
0.89	1.73	1.885	1.925	1.95	1.96	1.95
$F(z_1)$	1.000	1.434	1.806	2.369	2.740	3.142
$h = 0.06$						
0.77	1.58	1.68	1.725	1.74	1.74	1.74
0.80	1.59	1.70	1.74	1.755	1.76	1.76
0.83	1.60	1.72	1.75	1.77	1.78	1.78
0.86	1.61	1.73	1.77	1.79	1.79	1.80
0.89	1.63	1.74	1.79	1.805	1.81	1.81
$F(z_1)$	1.000	1.406	1.735	2.135	2.459	2.725

We thus find the relationships which we require $\frac{\overline{F(z_1)}}{F(z_1)}$ (see /186
Table 19).

Let us now determine the magnitude of K, i.e., the relationship between the illumination of a horizontal surface from the sky and the illumination of the same surface from the sun. This can be determined experimentally by the photometric method.

A surface covered with magnesium vapor is placed on the level on a high stand. At a small distance above it, a flat mirror is placed which transmits light which is reflected from the surface into the tube of a "diurnal sky photometer". Another photometer tube is directed, as is customary, toward the zenith. If desired, a small screen can throw shade from the sun onto the surface, leaving the rest of the sky exposed. A photometric wedge can be used to equalize the brightness of the field of vision sections which receive light from the zenith and from the horizontal surface, both when it is shaded and when it is fully illuminated. The

difference in the wedge readings is taken, and the desired ratio K is thus determined. The observations were carried out by a student at the Institute of Astronomy and Physics of the Kazakh SSR Academy of Sciences, M. G. Karimov, in an area at the Astronomical Observatory located on the outskirts of Alma-Ata, where the horizon was almost completely exposed. Green and red Schott filters with effective wavelengths 547 mμ and 625 mμ were employed.

The first filter corresponded closely to the maximum sensitivity of the eye.

TABLE 19

/187

$h = 0.04$						
$\begin{matrix} \text{sec } z \\ p \end{matrix}$	1.0	1.5	2	3	4	6
0.77	1.66	1.255	1.015	0.782	0.677	0.589
0.80	1.68	1.271	1.027	0.792	0.684	0.597
0.83	1.70	1.285	1.04	0.804	0.696	0.605
0.86	1.71	1.295	1.052	0.812	0.705	0.615
0.89	1.73	1.315	0.066	0.822	0.716	0.621
$h = 0.06$						
0.77	1.58	1.195	0.993	0.797	0.708	0.638
0.80	1.59	1.210	1.003	0.803	0.717	0.646
0.83	1.610	1.216	1.012	0.812	0.722	0.652
0.86	1.62	1.232	1.020	0.820	0.728	0.660
0.89	1.63	1.239	1.032	0.827	0.737	0.665

Similar observations were also carried out by A. P. Kutyreva in 1949 with a Yanishevskiy pyranometer having different filters. This pyranometer was systematically shaded, as was indicated above. The values of K , obtained by Kutyreva with a green filter for an unknown transmission coefficient of the atmosphere, were as follows:

sec z	1	1.5	2	3	4
K	0.08	0.12	0.16	0.24	0.32

Observations employing the Yanishevskiy pyranometer entailed the same principal disadvantage that light is thus recorded which passes through the surface layer of the device, and is not reflected

from it. In the case of transmitted light at large angles of incidence, the portion of energy influencing the thermal elements can be significantly reduced, and therefore the values of K can be invalid to a certain extent, particularly when the sun is low.

/188

On the other hand, in the case of a plate which reflects light according to the law of Lambert - which is exactly the case with a surface covered by magnesium vapor - the observed relationship between the brightnesses closely corresponds to the desired illumination relationship.

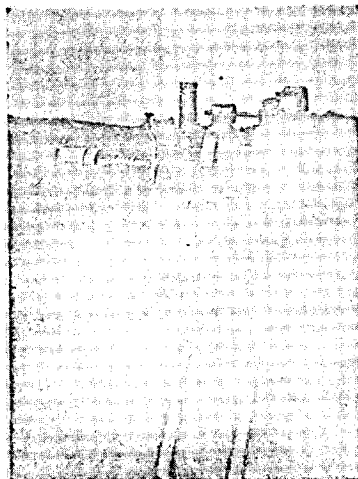
Due to the fact that I had the preliminary values of K which were obtained with the Yanishevskiy pyranometer, I examined the observations performed by Elvey (Ref. 70) which were carried out in different localities in America (Chile, Argentina, vicinity of Chicago, etc.) and also at the MacDonald Observatory in Texas. The purpose of these observations was to determine the effective height of the ionosphere layer luminescence.

For the observations Elvey employed a spectrograph with a collimator, which was 20" long, and with a Schmidt camera with a 2" aperture and with an aperture ratio of 1:0.66. The dispersion comprised 1500 Å/mm for the sodium line. Rectangular prisms were placed in front of the slit, which was 2" long. These prisms made it possible to expose sections of the sky simultaneously at zenith distances of 0°, 40°, 60°, 70°, 80° and 85°. A standard lamp which produced a spectrum through an additional prism was employed to control the transmission uniformity of sections of the spectrograph. The candlepower uniformity of the lamp was controlled by a photoelement with a barrier film. The latter comprises one shortcoming of the method, since the sensitivity of such a photoelement strongly depends on temperature, and the temperature must change during the night. A correction for the sensitivity of the photoelement was apparently not introduced. Elvey did not provide original data in his study. He reduced the observed brightnesses to the boundary of the atmosphere, making the formal assumption that it is satisfactory to employ an increased value of the transmission coefficient, which corresponds to an optical thickness which is two times smaller than the normal one, in order to determine both normal absorption and diffuse light.

Thus - as can be assumed - if the transmission coefficient during the observations of Elvey equals 0.85, which corresponds to an optical thickness of $k = -\ln p = 0.162$, then - according to this author - the following is assumed for reduction of the observations:

/189

$$k_1 = 0.081, \text{ i.e., } p_1 = 0.922.$$



Photometer of Night Sky

Our preceding discussion shows, however, that this is incorrect. /189
 If we base our argument on calculations derived from observations with the pyranometer, then for $p = 0.80$ we find that the correction for diffuse light assumes the following values:

TABLE 20

$h \backslash \sec z$	1	1.5	2	3	4
0.04	0.134	0.152	0.165	0.190	0.220
0.06	0.127	0.146	0.160	0.193	0.229

The entire reduction for the atmosphere is $p^{\sec z}(1 + X)$ (Table 21), i.e., it barely depends on h .

TABLE 21

$h \backslash \sec z$	1	1.5	2	3	4
0.04	0.907	0.824	0.745	0.609	0.500
0.06	0.902	0.820	0.742	0.611	0.504

Representing these values by the empirical expression $ap_1^{\sec z}$, we find the following for $a = 1.096$ and $p_1 = 0.823$:

$\sec z$	1	1.5	2	3	4
$ap_1^{\sec z}$	0.902	0.818	0.792	0.610	0.502

/190

This coincides quite closely with the preceding values. The constant a has no influence on the distribution of the ionosphere luminescence intensity as a function of z . It thus comes down to a certain increase in p , but not a significant increase - as was assumed by Elvey. If this is taken into account and if the observations of Elvey are used with a value of p which is not 0.922 but which is lower - namely, 0.88 - which is much more probable, then the following distribution of actual brightnesses at the boundary of the atmosphere is obtained for the green line 5577:

z	0°	40°	60°	70°	80°	85°
j	1.00	1.26	1.57	2.09	3.05	3.94

The following theoretical distribution, which is calculated for different heights of the effective layer (Table 22), corresponds to this.

TABLE 22

$h \backslash z$	0°	40°	60°	70°	80°	85°	h
0.04	1.00	1.27	1.80	2.34	3.10	3.46	255 km
0.05	1.00	1.26	1.77	2.25	2.89	3.16	319
0.06	1.00	1.26	1.74	2.16	2.71	2.92	382
0.08	1.00	1.18	1.67	2.04	2.43	2.59	510

As can be seen, the observations of Elvey are not reliably represented by a theoretical formula. Nevertheless, if great importance is attributed to the largest zenith distances, it can be assumed that h closely corresponds to a height on the order of 300 km, and in any case not 100 km, where the ionization layer E is located. Performing the reduction given above for atmosphere, Elvey himself concludes that h is on the order of 500 km. /191

Similar results were obtained from photometric observations carried out at Alma-Ata in 1945-1946. These observations were used, taking into account the more accurate values of the correction X , for the effect of diffuse light in the troposphere.

It is particularly important to derive the relationship between illumination on a horizontal surface from a clear sky and from the sun, at different zenith distances and different atmosphere transmission coefficients. The observations indicated above make it possible to formulate an adequate concept of this relationship, although the

presence of partial cloudiness, on the one hand, and light cirrus clouds on the other hand, introduces certain systematic errors. The drawings given below present the actual observations for certain days of the relationship between illumination on a horizontal surface from the sun and illumination from the entire celestial vault. Apparently, the relationship must equal zero for 90° . The observations were performed only in the morning hours, when the atmospheric transmission was most stable, and barely differed from its nocturnal value.

As can be seen, at zenith distances of the sun which are less than 60° , the atmospheric conditions are usually disturbed and the indicated relationship is less valid. Therefore, beginning with this value of z and up to the zenith, the curve in the graph was based on an approximative calculation, while the change in the inclination of the solar rays toward the plane of the horizon was primarily taken into account. Based on the observed curves, it is possible to draw analogous curves which correspond to equidistant values of p - namely, 0.77, 0.80, 0.83, 0.86 and 0.89.

The curves corresponding to extreme values were drawn by extrapolation, and are therefore approximate. All of these curves are shown in Figure 10.

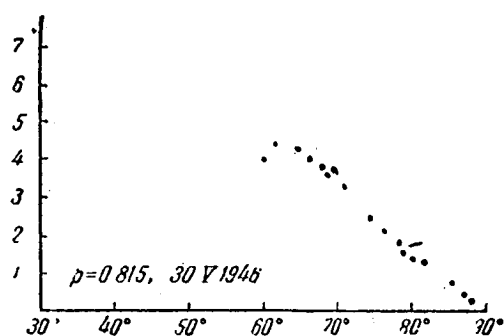
/193

Determining the ordinate values for different atmospheric masses and calculating their reciprocals K , we obtain the following Table.

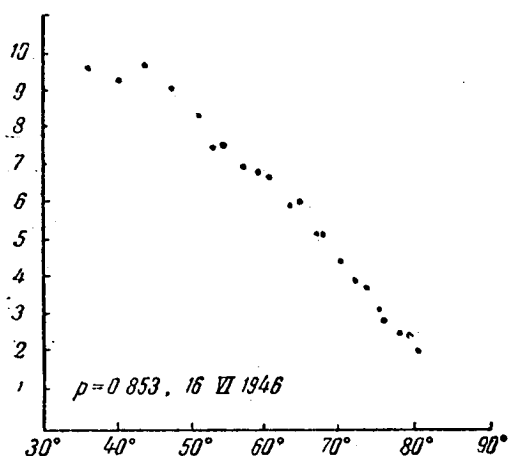
It is interesting to note that the values of K , which are presented above based on the observations of A. P. Kuttyreva with a Yanishevskiy pyranometer, almost exactly correspond to the data in this Table in conformity with the value $p = 0.86$, which is most frequently encountered under the mountainous conditions of Alma-Ata with an absolutely stable and clear sky.

TABLE 23
VALUES OF THE FUNCTION $K \left(\frac{\text{SKY-}}{\text{SUN}} \right)$

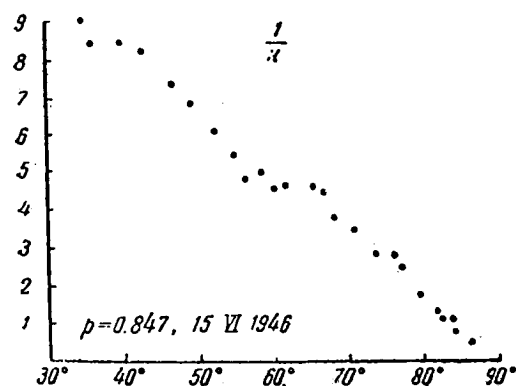
$\begin{matrix} \text{sec } z \\ p \end{matrix}$	1.0	1.5	2	3	4	6
0.77	0.123	0.185	0.259	0.400	0.556	0.909
0.80	0.109	0.164	0.222	0.345	0.470	0.769
0.83	0.092	0.139	0.185	0.294	0.400	0.667
0.86	0.078	0.115	0.154	0.238	0.345	0.588
0.89	0.069	0.099	0.130	0.208	0.294	0.500



5/30/1946



6/16/1946



6/15/1946

Figure 9.

Relationship Between Illumination on a Horizontal Surface from the Sun and from the Sky.

Using these data and the calculated relationship $\frac{F(z_1)}{F(z_1, A_1)}$, we can determine the values of X (see Table 24).

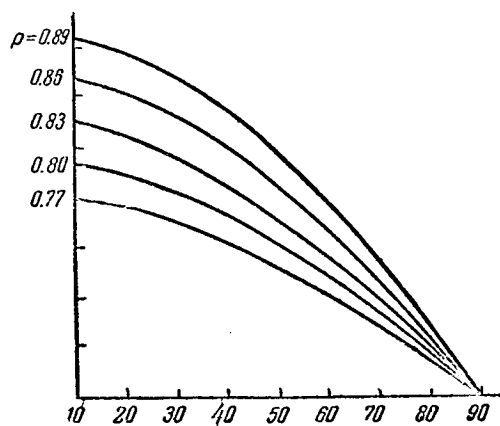


Figure 10

Smoothed Values for the Relationship Between
Illumination on a Horizontal Surface from the
Sun and from the Sky.

TABLE 24

$p \backslash \sec z$	1.0	1.5	2	3	4	6
$h=0.04$						
0.77	0.204	0.132	0.254	0.313	0.376	0.534
0.80	0.183	0.109	0.227	0.273	0.326	0.459
0.83	0.156	0.178	0.192	0.236	0.278	0.403
0.86	0.133	0.149	0.162	0.194	0.243	0.361
0.89	0.119	0.130	0.138	0.171	0.210	0.310
$h=0.06$						
0.77	0.194	0.221	0.249	0.318	0.394	0.580
0.80	0.173	0.198	0.222	0.277	0.342	0.495
0.83	0.148	0.169	0.187	0.238	0.288	0.434
0.86	0.126	0.142	0.157	0.195	0.251	0.387
0.89	0.112	0.123	0.134	0.172	0.216	0.332

/194

Thus, according to our theory, reduction for the effect of
atmosphere during observations of ionosphere brightness is expressed
by the factor

$$p^{\sec z}(1 + X).$$

Let us try to represent this factor in a simplified manner, using the empirical expression $ap_1^{\sec z}$, so that for each value we must have

$$p^{\sec z} (1 + X) = ap_1^{\sec z}$$

From an equation of the following type

$$\lg a + \sec z \lg \frac{p_1}{p} = \lg (1 + X)$$

we can find the unknowns $\lg a$ and $\lg \frac{p_1}{p}$ according to the method of least squares. In order to obtain the correct brightness distribution over the celestial vault, it is sufficient to know one p_1 . /195

The results can be represented by the following Table:

TABLE 25

p	h=0.04		h=0.06	
	p ₁	Δp	p ₁	Δp
0.77	0.808	0.04	0.814	0.04
0.80	0.829	0.03	0.839	0.04
0.83	0.858	0.03	0.863	0.03
0.86	0.889	0.03	0.889	0.03
0.89	0.919	0.03	0.923	0.03

We thus find a very simple method for determining the effect of diffuse tropospheric light on the brightness distribution of the ionosphere as a function of atmospheric mass: the transmission coefficient must only be increased by 0.03, if the atmosphere has normal transmission - i.e., if $p > 0.80$. Representing the quantities $p^{\sec z}(1 + X)$ by the expression $a(p + 0.03)^{\sec z}$, we find the following values for the constant a .

TABLE 26

h=0.04						h=0.06				
sec z \ p	0.77	0.80	0.83	0.86	0.89	0.77	0.80	0.83	0.86	0.89
1	1.16	1.14	1.12	1.09	1.08	1.15	1.13	1.11	1.09	1.08
1.5	1.16	1.14	1.12	1.09	1.08	1.15	1.13	1.11	1.08	1.07
2	1.19	1.14	1.11	1.08	1.07	1.16	1.14	1.11	1.08	1.06
3	1.17	1.14	1.11	1.08	1.06	1.17	1.14	1.11	1.08	1.06
4	1.18	1.15	1.11	1.08	1.06	1.18	1.16	1.12	1.09	1.07
6	1.22	1.17	1.13	1.11	1.07	1.24	1.0	1.16	1.13	1.09
mean	1.18	1.15	1.12	1.09	1.07	1.17	1.15	1.12	1.09	1.07

Thus, α changes slightly as a function of p , but for a given state of the atmosphere it remains constant, within the limits of observational accuracy, for all zenith distances up to 80° .

For normal atmospheric transmission, it can be assumed that in every case the factor contained in the expression for apparent atmospheric brightness is

$$1.1 (p + 0.03)^{\sec z}.$$

To a significant extent, the problem of illumination of the troposphere by the ionosphere was analyzed above on the basis of theoretical considerations. In order to verify the validity of these considerations, let us examine a simplified problem representing the possibility of comparing results with observations. Let the ionosphere, above the level of the observer, have identical brightness at every point for all possible zenith distances. The brightness of the troposphere, as compared with the observed brightness of the ionosphere, will equal the relationship between the illumination on a horizontal surface from the sky and the illumination from the sun - as is already known.

From the theoretical point of view, this tropospheric brightness can be represented by the expression

$$J = \int L \frac{\mu}{k} f(\vartheta) \varphi(z, z_1) \sec z_1 d\sigma$$

or in the given case

$$J = L \frac{\mu}{k} \sec z_1 \int f(\vartheta) \varphi(z, z_1) d\sigma,$$

where L is the brightness of the ionosphere; μ , k are the coefficients of the scattering and absorption; $f(\vartheta)$ is the scattering indicatrix; and z is the zenith distance of the observed point in the sky.

The function

$$\varphi(z, z_1) = \frac{\rho^{\sec z} - \rho^{\sec z_1}}{\sec z_1 - \sec z}$$

changes very slightly as a function of z . Its mean value corresponds to approximately $z = 60^\circ$. Taking the average and taking it out of the integral sign, we have

$$J = L \frac{\mu}{k} \sec z_1 \overline{\varphi(z, z_1)} \int f(\vartheta) d\sigma.$$

The integral

$$\int f(\vartheta) d\sigma = \int_0^{\frac{\pi}{2}} \int_0^{2\pi} (1 + a_2 \cos^2 \vartheta + a_3 \cos^3 \vartheta) \sin z dz dA,$$

where

$$\cos \vartheta = \cos z_1 \cos z_1 + \sin z_1 \sin z_1 \cos A,$$

can be readily calculated.

Let us take the fact into account that

$$k = 2\pi\mu \int_0^{\pi} (1 + a_2 \cos^2 \vartheta + a_3 \cos^3 \vartheta) \sin \vartheta d\vartheta$$

or

$$\frac{\mu}{k} = \frac{1}{4\pi \left(1 + \frac{a_2}{3} + \frac{a_3}{4}\right)},$$

where

$$a_2 = 1.652; \quad a_3 = 1.214.$$

We find that

$$J = L \overline{\varphi(z, z_1)} 0.417 (1 + 0.3 \cos z_1 - 0.1 \cos^3 z_1) \sec z_1.$$

It can be assumed that the mean values $\overline{\varphi(z, z_1)}$ are as follows for $p = 0.83$:

$\sec z_1$	1	2	3	4	6
$\overline{\varphi(z, z_1)}$	0.14	0.13	0.12	0.11	0.09

or

$$\overline{\varphi(z, z_1)} = 0.15 - 0.01 \sec z_1.$$

Thus, in this case the relationship between the troposphere brightness and the observed ionosphere brightness will be as follows: /198

$$\frac{I}{L p^{\sec z_1}} = \frac{0.0625}{p^{\sec z_1}} (1 + 0.3 \cos z_1 - 0.1 \cos^3 z_1) \left(1 - \frac{1}{15} \sec z_1\right) \sec z_1.$$

The theoretical and observed values for this illumination relationship can be compared in the following Table:

$\sec z_1$	1	2	3	4	6
calc. . .	0.084	0.179	0.288	0.413	0.72
obs. . .	0.09	0.19	0.29	0.40	0.66

The agreement is quite good, taking the fact into account that the calculation was performed with different simplifying assumptions,

and the observations did not pertain to ideally clear weather. This simple calculation convinces us once again that our theoretical formulations correspond to reality.

Let us now deal with the problem of troposphere illumination by all the stars comprising our galaxy. Let us determine the integrated brightness of all the stars as a function of the galactic latitude, expressed in the number of stars of the fifth magnitude per square degree. For this purpose, let us employ the calculations of Searces (Ref. 71) which give the number of stars up to the m^{th} apparent magnitude ($m \leq 20$). The desired integrated brightness ranging from m up to ∞ is:

$$J_m^\infty = \int_m^\infty \frac{dN_m}{dm} \cdot 2.512^{5-m} dm.$$

Because the data given by Searces pertain to photographic quantities, a correction must be introduced for the color index for reduction to visual quantities. According to Searces, for the entire sky we have

/199

$$C.I. = 0.16 + 0.050 m.$$

In order to obtain the distribution of a number of stars with respect to apparent magnitude, it is sufficient to only add the value C.I. algebraically to the stellar magnitudes of Searces, leaving a number of stars unchanged. Interpolating a number of stars with respect to the constant interval Δm and calculating the derivatives $\frac{dN_m}{dm}$, by numerical integration we find the values of J_m^∞ .

For different m and galactic latitudes b , we have:

TABLE 27

$m \backslash b$	15°	30°	45°	60°	75°	90°	105°	120°	135°	150°
7-8	0.082	0.070	0.052	0.054	0.044	0.040	0.036	0.034	0.030	0.030
8-10	0.193	0.162	0.140	0.123	0.104	0.093	0.080	0.072	0.064	0.064
10-12	0.216	0.184	0.156	0.154	0.106	0.086	0.072	0.064	0.056	0.054
12-14	0.230	0.174	0.142	0.120	0.086	0.068	0.05	0.044	0.040	0.038
14-16	0.191	0.136	0.102	0.090	0.054	0.040	0.030	0.024	0.020	0.020
16-18	0.130	0.074	0.056	0.040	0.026	0.016	0.012	0.010	0.010	0.008
18-20	0.068	0.034	0.022	0.016	0.008	0.00	0.004	0.003	0.00	0.002
7-20	1.002	0.848	0.680	0.572	0.428	0.350	0.288	0.250	0.224	0.208
7- ∞	1.136	0.865	0.691	0.580	0.432	0.353	0.292	0.252	0.225	0.209

It must be pointed out that these results pertain to the average

stellar distribution which is characteristic for a typical stellar universe. At individual locations in the sky, the integrated brightness of the stars can differ more or less significantly from the average which corresponds to the galactic latitude. It is particularly important to note that the apparent defect of the stellar density in the region of the north pole - which, as is known, is covered by dark nebulae - is barely visible to the naked eye. A great many studies have been carried out (Ref. 72) on the defect of stellar density around the north pole. The following defects can be assumed in a number of stars having different magnitudes:

/200

m	7	8	9	10	11	12
$\Delta \lg N_m$	-0.02	-0.035	-0.03	-0.17	-0.18	-0.17
m	13	14	15	16	17	18
$\Delta \lg N_m$	-0.31	-0.33	-0.26	-0.20	-0.16	-0.19

If this is taken into account, we find that the integrated brightness of stars toward the pole equals 0.399 of our units - i.e., it is smaller by a factor of 0.18 than the mean value assumed for the same galactic latitude = 30° . This conclusion is of significant importance, because the region of the sky around the pole is used as a standard in studying the radiance of the nocturnal sky.

The integrated brightness of brighter stars - for example, from $m = 4$ to $m = 7$ - is small.

This quantity can be determined on the basis of the same tables given by Searces, using the approximate formula

$$I_1 = \sum 2.512^{5-m} \frac{N_m}{\lg e} \left(\lg N_{m+\frac{1}{2}} - \lg N_{m-\frac{1}{2}} \right).$$

Using the trapezoid method, we find the following as a result of numerical calculations:

b	J'_1	b	J'_1
0°	0.254	40°	0.098
20°	0.158	90°	0.073

Let us examine the effect of the galaxy on the radiance of the /201 troposphere.

Let us assume that the Milky Way passes through the zenith of the observational point, so that the galactic pole lies on the horizon

or is close to it. Let the integrated brightness of all the stars - which is expressed, for example, in the number of stars of the fifth magnitude per square degree - be $L(b)$, as was found above.

This function can be represented with the aid of the following simple formula

$$L(b) = C - C_1 \sin b + C_2 \sin^2 b,$$

where

$$C = 1.92; \quad C_1 = 3.42; \quad C_2 = \frac{C_1}{2}.$$

The observed integrated brightness of the stars can be represented by this formula in the following way:

b	0°	15°	30°	60°	90°
$L(b)$ calc.	1.92	1.14	0.64	0.24	0.21
$L(b)$ obs.	1.8	1.14	0.58	0.29	0.21

The brightness of the sky at a certain point with the zenith distance z_1 is

$$J(z_1) = \int L(b) d\sigma \frac{\mu}{k} f(\vartheta) \varphi(z, z_1) \sec z_1$$

or, averaging $\varphi(z, z_1)$, we have

$$J(z_1) = \frac{\mu}{k} \overline{\varphi(z, z_1)} \sec z_1 \int L(b) d\sigma f(\vartheta).$$

The scattering indicatrix $f(\vartheta)$ is used in the normal form

$$f(\vartheta) = 1 + a_2 \cos^2 \vartheta + a_3 \cos^3 \vartheta,$$

where

$$\cos \vartheta = \cos z \cos \sin z \sin z_1 \cos A$$

or

$$\cos \vartheta = \sin b \sin b_1 + \cos b \cos b_1 \cos l.$$

z, A designate the horizontal coordinates, and b, l designate the galactic coordinates. In the special case under consideration, it is advantageous to perform integration within the limits of $b = -\frac{\pi}{2}$ to $b = \frac{\pi}{2}$.

Since

$$d\sigma = \cos b db dl,$$

the problem can be reduced to calculating the simple integral

$$J(z_1) = \frac{\mu}{k} \overline{\varphi(z, z_1)} \sec z_1 \pi \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \left(C - C_1 \sin b + \frac{C_1}{2} \sin^2 b \right) \cos b db \times \\ \times \left[1 + a_2 \left(\sin^2 b \sin^2 b_1 + \cos^2 b \cos^2 b_1 \cdot \frac{1}{2} \right) + \right. \\ \left. + a_3 \left(\sin^3 b \sin^3 b_1 + \frac{3}{2} \sin b \sin b_1 \cos^2 b \cos^2 b_1 \right) \right].$$

It must be noted that the sign before $\sin b$ must be reversed when performing integration from $-\frac{\pi}{2}$ to 0.

After performing integration, we find the expression

$$J(z_1) = \frac{\mu}{k} \overline{\varphi(z, z_1)} \pi \sec z_1 \left[2C \left(1 + \frac{a_2}{3} \right) - \right. \\ \left. - \frac{C_1}{3} \left(2 + \frac{11a_2}{20} \right) - C_1 a_2 \sin^2 b_1 \frac{7}{60} \right].$$

Because

$$\frac{\mu}{k} = \frac{1}{4\pi \left(1 + \frac{a_2}{3} + \frac{a_3}{4} \right)}; \quad a_2 = 1.652; \quad a_3 = 1.214,$$

we obtain

$$J(z_1) = 0.355 \overline{\varphi(z, z_1)} \sec z_1 \left(1 - \frac{1}{4} \sin^2 b_1 \right).$$

We would like to point out that observations of more than six atmospheric masses should not be carried out, since close to the horizon it is usually impossible to guarantee uniformity in the optical properties of the atmosphere, due to the harmful influence of the surface layer. If the mean value of $\overline{\varphi(z, z_1)}$ is determined from the expression

$$\overline{\varphi(z, z_1)} = \frac{\int \int \varphi(z, z_1) \sin z dz dA}{\int \int \sin z dz dA},$$

then there is very close agreement between this mean value and $\varphi(z, 60^\circ)$. On the basis of this, we obtained the following value of $\varphi(z, 60^\circ)$ for different transmission coefficients p .

/203

TABLE 28

p \ sec z ₁	$\varphi(z, \cos^2)$					
	1	1.5	2	3	4	6
0.77	0.177	0.166	0.135	0.137	0.120	0.096
0.80	0.160	0.152	0.143	0.123	0.115	0.094
0.83	0.141	0.136	0.138	0.117	0.107	0.090
0.86	0.120	0.116	0.111	0.104	0.096	0.084
0.89	0.093	0.096	0.0925	0.078	0.072	0.074

TABLE 29

p \ sec z ₁	0.355 $\varphi(z, z_1) \sec z_1$					
	1	1.5	2	3	4	6
0.77	0.063	0.088	0.110	0.146	0.170	0.204
0.80	0.057	0.081	0.101	0.132	0.163	0.200
0.83	0.050	0.072	0.091	0.124	0.152	0.191
0.86	0.042	0.062	0.079	0.110	0.136	0.179
0.89	0.035	0.051	0.066	0.094	0.116	0.158

Thus, the main term in the expression for the troposphere component of galactic airglow is as shown above (see Table 29).

/204

As can thus be seen, for normal atmospheric transmission the correction for scattered galactic light only rarely exceeds 0.15 in our units.

Let us also perform similar calculations in a simplified scheme, in which

$$L(b) = C - C_1 \sin b \text{ и } f(\vartheta) = 1 + a_1 \cos \vartheta.$$

In this case, the problem can be reduced to calculating the integral

$$J(z_1) = \frac{\mu}{k} \overline{\varphi(z, z_1)} \sec z_1 \pi \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (C - C_1 \sin b)(1 + a \sin b \sin b_1) \cos b db.$$

As a result, we find

$$J(z_1) = \frac{\mu}{k} \overline{\varphi(z, z_1)} \sec z_1 \pi (2C - C_1).$$

In this case, no dependence on b_1 can be determined. It is

interesting to note that $J(z_1)$ does not depend on a_1 and a_3 , i.e., on the asymmetry of the scattering indicatrix.

Let us examine another extreme case, when the galactic plane coincides with the horizon. In this case, the brightness of the troposphere at the zenith distance z can be represented with the same approximation by the expression

$$J(z_1) = \frac{\mu}{k} \sec z_1 \overline{\varphi(z, z_1)} \int L(b) f(\vartheta) d\sigma,$$

and

$$d\sigma = \sin z dz dA; \quad L(b) = C - C_1 \cos z + \frac{C_1}{2} \cos^2 z,$$

$$f(\vartheta) = 1 + a_2 \cos^2 \vartheta + a_3 \cos^3 \vartheta,$$

with the same values of C, C_1, a_2, a_3 .

Calculating the integral, we obtain

/205

$$\begin{aligned} J(z_1) = & \frac{\mu}{k} \sec z_1 \overline{\varphi(z, z_1)} 2\pi \left[C - \frac{C_1}{3} + a_2 \left(\frac{7C}{16} - C_1 \frac{11}{120} \right) + \right. \\ & + \frac{3}{8} a_3 \left(C - \frac{1}{5} C_1 \right) \cos z_1 - \left(\frac{5}{48} C - \frac{C_1}{120} \right) a_2 \cos^2 z_1 - \\ & \left. - \frac{1}{8} a_3 \left(C + \frac{1}{3} C_1 \right) \cos^3 z_1 \right]. \end{aligned}$$

Substituting numerical values, we find

$$\begin{aligned} J(z_1) = & \frac{\mu}{k} \sec z_1 \overline{\varphi(z, z_1)} 2\pi [1.68 + 0.56 \cos z_1 - \\ & - 0.28 \cos^2 z_1 - 0.46 \cos^3 z_1] \end{aligned}$$

or, finally

$$J(z_1) = \frac{\overline{\varphi(z, z_1)}}{3.7} [1.68 + 0.56 \cos z_1 - 0.28 \cos^2 z_1 - 0.46 \cos^3 z_1] \sec z_1.$$

For different values of z_1 and for $p = 0.86$, this yields the following:

$\sec z_1$	1	1.5	2	3	4	6
$J \dots$	0.048	0.037	0.110	0.151	0.185	0.239

If these numbers are compared with the data given in the preceding Table for $b_1 = 0$, then it can be stated that the dependence

with zenith distance is approximately the same, as well as the value for the troposphere brightness. In the general case, when the Milky Way is inclined toward the horizon, a solution of the problem entails a great many difficulties. However, because the results differ very little from each other for extreme cases, from this point on we shall use the tables which were compiled for aligning the Milky Way in a normal direction to the horizon.

We must now examine the effect of the zodiacal component on the troposphere. We would like to point out that the brightness of the zodiacal light increases extremely rapidly as the sun is approached. /206 Therefore, the brightest sections of the zodiacal light, which exceed the brightness of the Milky Way, can be observed for only a brief period of time shortly before sunrise or just after sunset. In usual photometric observations of zodiacal light, the observer concentrates his attention on a specific almucantar - for example, about 75° of the zenith distance. Different portions of the zodiacal light are observed at this zenith distance leading away from the horizon. This method has many advantages. In the first place, atmospheric absorption and the ionosphere component remain approximately the same. Therefore, independently of their objective value, the observed brightnesses represent the isophote distribution of the zodiacal light, and correctly characterize their form and distribution. In addition, the dim portions of zodiacal light are observed when the brighter regions are still concealed behind the horizon and, consequently, cannot have a distorting influence due to light scattering in the troposphere. Finally, these bright portions, which are closest to the sun, can be observed with a maximum savings in time. During the night the observer is only concerned with the dim zodiacal component, which - as can be assumed - changes very little in the celestial vault, and produces a significant zodiacal band only in the ecliptic plane. Thus, it can be assumed that the zodiacal component has constant brightness in almost every case when one is calculating the light scattering in the troposphere. In this case, the additional factor for troposphere scattering, which is used to correct the brightness of zodiacal light, is simply represented by the function K , obtained from observations on the illumination of a horizontal surface from the sky and from the sun. The total observed brightness of the zodiacal component (LZ) will therefore be

$$(LZ) p^{\sec z} (1+K).$$

Thus, the brightness of the nocturnal sky (not of zodiacal light) /207 when there is absolutely no twilight can be represented by the following formula, taking into account both the primary and the secondary factors

$$J = \frac{C(1+h) p^{\sec z} (1+K)}{\sqrt{(1+h)^2 - \sin^2 z}} + L(b_1) p^{\sec z} + 0.355 \overline{\varphi(z, z_1)} \sec z_1 \times \\ \times \left(1 - \frac{1}{4} \sin^2 b_1 \right) + (LZ) p^{\sec z} (1+K)$$

or with sufficient accuracy

$$J = \frac{C_1 (1 + h) (p + 0.03)^{\sec z}}{\sqrt{(1 + h)^2 - \sin^2 z}} + L (b_1) p^{\sec z} +$$

$$+ 0.355 \sqrt{\varphi(z, z_1)} \sec z_1 \left(1 - \frac{1}{4} \sin^2 b_1 \right) + (LZ) p^{\sec z} (1 + K).$$

CHAPTER 13

Determination of the Effective Height of Ionosphere Airglow

Let us employ the theory which was developed previously to determine the effective height of ionosphere airglow. For this purpose, I observed the brightness of the nocturnal sky at different zenith distances in the western and eastern portions of the first vertical, using a photometer which was constructed for this purpose. The photometer had the most simple construction, and was advantageous for nocturnal observations. A cylinder, whose head was covered by a compound having constant illumination made of zinc sulfide, with an admixture of radioactive substance, moved within a cylindrical tube. This tube was about 20 cm long and about 4 cm wide, and was mounted on an azimuthal stand with a level and graded circles. A dull scattering screen, which was illuminated by the fluorescent head of the moving cylinder, was mounted on the inner surface of the lid, which was slipped onto the forward part of the tube. Next to the screen an aperture was made having the same form and dimensions, through which it was possible to see a small section of the nocturnal sky through a given portion of the tube. For this purpose, a prism was mounted on the tube having total inner reflection - the only optical element in the entire apparatus - and an aperture was made in the center of the illuminating cylinder, in order to clearly see the scattering screen in any position.

/208

The brightness of the dull screen changed as the cylinder moved, and was equated with the brightness of the observed section of the sky, which was recorded on a strip of heavy paper laid on a rule along the outer generatrix of the tube. The recording device was constructed in such a way that it was possible to observe several objects having different brightness, without confusing them. The tube was adjusted with the aid of a small selector or by means of circles which were divided every 5°. The dividing lines on the circles were lined with a luminescent substance which could be clearly seen in the dark. Thus, all the observations could be performed in total darkness, retaining the adaptation of the eye for maximum pupil dimensions. A lamp was employed for the necessary recordings, which had similar luminescence and which was bright enough for orientation in the dark, for replacing the strip of paper, for reading the time, etc.

/209

The same lamp could be used advantageously for standardization of the photometer - i.e., to express its scale readings in absolute units, which we used - namely, in the number of stars of the fifth magnitude per square degree. For this purpose, a lid was slipped on to the lamp having a small circular opening which was approximately

2 mm in diameter. At a distance of several meters, this opening had an actual star-shaped form and could be readily compared in terms of brightness with different stars having a known brightness. It was advisable to paste a piece of white paper onto the lamp lid, in order that the artificial star would be represented on the background with approximately the same brightness as the reference star. On the other hand, if the lid with the opening were removed, it was possible to determine the brightness of the lamp base which provides uniform illumination, using the photometer described above. Due to the significant brightness of the lamp, it was necessary to employ a Talbot disc, which decreased the brightness by several factors. Thus, this lamp made it possible to employ both a point object, comparing it with the stars, and a surface object, observing it with the aid of the photometer described above. A comparison of these observations made it possible /210 to express the photometer scale in the absolute units given above.

Due to considerable reflection from the inner tube walls, there was thus no possibility of determining the brightness change in the photometer scale in order to apply the inverse square law.

In actuality, the photometer scale is much flatter, and is almost exactly linear. Therefore, this scale was calibrated entirely using a Nagel adaptometer in the optical laboratory of the Institute of Astronomy and Physics of the Kazakh SSR Academy of Sciences.

Under the program which I had formulated, junior scientific associate of the Institute of Astronomy and Physics, N. B. Divari, used this instrument to perform observations on the mountain Laptev Kurgan, which was 1800 m high and was 10 km east of the village of Talgar (Alma-Atinskaya Region), and also on the Kamenskiy Plateau which was 10 km south of Alma Ata at an altitude of 1350 m. Unfortunately, poor weather and unusually violent disturbances obstructed the actual uniformity of the ionosphere, and made the July observations invalid to a certain extent. In August, conditions which were much more stable in every respect were established, and several tens of series to determine the brightness of the nocturnal sky were carried out at different zenith distances, as far as possible to the side of the Milky Way, and also at the western or eastern vertical. By calculating the galactic coordinates of the observational point, I was able to determine the effect of the galactic component in the number of stars of the fifth magnitude per square degree. As a precaution, only those observational series for which the galactic latitudes were not less than 30° were used. The effect of the zodiacal components at the points in the sky being studied could be disregarded. Thus, the brightness distribution at different zenith distances, which was caused by only the presence of the ionosphere, could be readily determined. Comparing this distribution with the theoretical curves corresponding to different

effective heights, we could find the curve which corresponded most closely to the observations. Tables 30 and 31 present the observations of N. B. Divari and their reductions (the mean value from the series is given), as well as the derived values of h expressed, as is usual, in units of the earth's radius.

/211

TABLE 30

21 VII		26 VII				27 VII		
z	j	z	j_{app}	red	j	j_{app}	red	j
18°	0.53	18°	1.09	0.55	0.54	1.16	0.44	0.72
38	1.03	28	1.20	0.43	0.77	1.30	0.35	0.95
58	1.21	38	1.45	0.36	1.09	1.41	0.30	1.11
68	1.54	48	1.62	0.31	1.31	1.60	0.27	1.33
78	1.68	58	1.78	0.28	1.50	1.78	0.25	1.53
		68	2.02	0.26	1.76	2.02	0.25	1.77
		78	2.22	0.26	1.96	2.02	0.24	1.78
$h=0.04$		$h=0.035$				$h=0.04$		

TABLE 31

30 VIII				31 VIII			
z	j_{app}	red	j	z	j_{app}	red	j
18.5°	1.55	0.48	1.07	—	—	—	—
28.5	1.78	0.38	1.40	28.5°	1.79	0.46	1.33
38.5	1.62	0.32	1.30	38.5	1.81	0.37	1.44
48.5	2.00	0.28	1.72	48.5	1.92	0.32	1.60
58.5	2.23	0.25	1.98	58.5	2.06	0.28	1.78
68.5	2.33	0.25	2.08	68.5	2.28	0.27	2.01
78.5	2.49	0.25	2.24	78.5	2.30	0.26	2.04
$h=0.05?$				$h=0.06$			

[z designates the zenith distance of the observed point of the first vertical; j_{app} the apparent brightness in the number of stars of the fifth magnitude to one square degree; red - correction for the galactic component; j - actual brightness of the ionosphere in relative units.] On the average, we find that $h = 0.044$, which corresponds to 280 km. Analogous observations on the brightness distribution at different zenith distances in individual wavelengths were carried out with a nebular spectrograph by a student at the Institute of Astronomy and Physics of the Kazakh SSR Academy of Sciences,

/212

M. G. Karimov. This nebular spectrograph, which had a Leontovskiy design, was described in detail in the Bulletin of the Astronomical Institute (now the Institute of Theoretical Astronomy of the USSR Academy of Sciences) (Ref. 73). The collimator with a long slit had an aperture ratio of 1/6; the camera with a Schmidt optical system had an aperture ratio of 1:1. A prism system was placed in front of the slit, which made it possible to photograph the spectrum of the nocturnal sky at five points simultaneously at different zenith distances, from the zenith to 78°. The exposure was made on Isopan SS or Isopan F films, and usually lasted for about 10 hours. The emission lines 5577, 6300, 6330, and the sodium line 5892, which was somewhat weaker, clearly appeared during this time. The lines in the more refracted portion of the spectrum could not be distinguished. A continuous spectrum was barely apparent, which slightly increased the over-all background of the film.

The observer usually followed the weather conditions, and confined himself to exposures when there was a completely clear sky. The transmission coefficient of the atmosphere was determined along with this, using the method of Pickering. Due to the fact that this method was based on visual estimates, it could not provide accurate results. Therefore the atmospheric transmission was determined using a halo photometer during the morning hours, after the nocturnal observation, as was indicated above. It was quite simple to derive a theoretical interpretation of such observations, since the galactic and zodiacal components had almost no effect on the emission lines. This effect was only manifested in a slight amplification of the continuous spectrum located at the boundary of distinguishability. We can thus /213 confine ourselves to employing only the first term in our formula for the total brightness of the nocturnal sky. The heterogeneity of the ionosphere structure could not be of significant importance, due to the long exposure time. Regular changes over a period of time were also excluded, since all regions of the sky were photographed simultaneously. I have presented below a summary of the logarithms determined by Karimov for the brightness of the emission line 5577 for different zenith distances.

TABLE 32

Date	p	p_1	0°	45°	60°	70°	75°	78°	h
1945 26/XI	0.80	0.84	0.00	—	0.170	—	0.215	—	0.040
1945 28/XII	0.81	0.84	0.00	—	0.193	—	0.220	—	0.040
1946 26/II	0.85	0.88	0.00	—	0.199	—	0.250	—	0.043
1946 23/VI	0.85	0.88	0.00	—	0.196	—	0.255	—	0.042
1946 30/VII	0.85	0.88	0.00	0.107	0.176	0.217	0.250	—	0.047
1946 22/VIII	0.82	0.85	0.00	0.093	0.164	0.218	—	0.210	0.044
1946 24/VIII	0.87	0.90	0.00	0.121	0.210	0.276	—	0.310	0.040
1946 1—2/IX	0.84— 0.85	0.87— 0.88	0.00	0.107	0.155	0.255	—	0.258	0.040
1946 4—5/IX	0.82?	0.85	0.00	0.086	0.164	0.176	—	0.199	0.05?

On the average, $h = 0.043$, which corresponds to an altitude of 270 km.

Thus, both computational methods produce practically the same height which is effective for the luminescent ionosphere layer. In spite of the fact that these determinations do not pretend to be very accurate, it can be stated with complete certainty that the layer E has no relationship to the night sky airglow, and that the luminescent layer is located at the height of the ionization layer F.

CHAPTER 14

Separation of the Components Comprising the Nocturnal Sky Luminosity

A simple survey of the nocturnal sky by the naked eye shows that /214 the luminosity of the celestial vault is caused by three different components: the earth's atmosphere (whose brightness increases toward the plane of the horizon, due to an increase in the atmospheric mass), the total brightness of the stars comprising our galactic system, and the zodiacal light (whose plane of symmetry almost coincides with the ecliptic). Under good atmospheric conditions and when the Milky Way is located at a high point in the sky, it can be readily observed that the brightness of the nocturnal sky gradually decreases at both of its sides. Bright galactic clouds can be clearly distinguished on the dark background of the southern sky, but even under the most favorable conditions their brightness exceeds the region of the sky around the North Pole by not more than 2 - 3 times. Such a comparatively small degree of contrast for the Milky Way depends on the fact that very intense ionosphere luminosity as well as its light, which is scattered into the lower atmospheric layers, are superimposed on it. Generally speaking, the celestial regions around the zenith appear to be the darkest. The brightness of the sky increases considerably as the horizon is approached, so that at a zenith distance of 80° the brightness is approximately two times greater than at the zenith. Finally, it is usually possible to see the zodiacal light in one or another form - sometimes in the form of rather bright cones - in the east before sunrise, or in the west shortly after sunset - or in the form of faint, diffused luminosity in the northern portion of the sky which combines both cones in the direction toward /215 the sun. It may also appear in the form of a barely distinguishable, blurred vacant area intersecting the southern portion of the sky along the zodiacal constellations. Each of these components extends throughout the entire sky, as would be expected. The phenomenon is complicated by the scattering of light into the lower atmospheric layers - the troposphere.

In order to determine the actual brightness and nature of the zodiacal light isophotes, it is necessary to determine the ionosphere and galactic components for any region of the sky, and to subtract them from the total observed brightness. The remainder represents the desired zodiacal airglow, which must be corrected for the effect of atmosphere. Generally speaking, it can be assumed that the zodiacal component is only significant at small angular distances from the sun, and primarily in the ecliptic plane.

In the celestial regions which are far removed from the sun,

zodiacal airglow is, in all probability, very faint, and in any case rather uniform. This can be seen from the fact that the brightness of the sky is almost exactly the same under very diverse conditions, at the same zenith distance and approximately at the same galactic latitude.

The problem of separating the components of ionosphere airglow at any point in the sky can be fully solved, if it can be solved for any one point which is most favorable for the celestial pole, where the zenith distance and the stellar configuration are always the same. The method which I employed for this purpose is as follows.

Let us assume that the region of the Milky Way, which has a fairly uniform background, and an adjacent region with a dark nebula are observed simultaneously. For this purpose, it is most advantageous to take the bright stellar cloud in the Scutum Sct. constellation, which can be observed within wide limits of zenith distances. Let us determine the brightness of this cloud Scut. Lumin., the dark nebula next to it Scut. Dark, and the region around the celestial pole (Polaris) simultaneously.

If the atmosphere has no airglow, and only light is absorbed, then the degree of contrast between the light and dark portions of the galaxy would not change. /216

In actuality, the atmosphere is superimposed on the stellar clouds of the Milky Way in the form of a luminescent veil, and reduces the contrast - the greater the corresponding zenith distance, the greater is the contrast reduction.

If there is zodiacal light in this direction, its role can be reduced to only increasing the component of cosmic airglow.

It was shown above that the observed brightness of the nocturnal sky can be represented by the formula

$$J = \frac{C(1+h)p_1^{\sec z}}{\sqrt{(1+h)^2 - \sin^2 z}} + j_{gal}p^{\sec z} + red(b, z) + LZp^{\sec z}(1+K).$$

For the region of the Milky Way, where $b = 0$, we have - assuming that

$$j_{gal} + LZ = j$$

(j represents the total cosmic component),

$$J = p j^{\sec z} + \frac{C(1+h)(p+0.03)^{\sec z}}{\sqrt{(1+h)^2 - \sin^2 z}} + red(b, z) + LZp^{\sec z} \cdot K.$$

In the first approximation, the zodiacal airglow which is scattered by the troposphere can be disregarded - i.e., $LZp^{\sec z} K$.

Assuming for the reduction

$$F(z) = \frac{1+h}{\sqrt{(1+h)^2 - \sin^2 z}},$$

we have

$$J = jp^{\sec z} + CF(z)(p + 0.03)^{\sec z} + \text{red}(b, z).$$

Employing this expression for Scut. Lumin. (j_1) and Scut Dark (j_2), which are located at the same zenith distance z , and subtracting one from the other, we find

$$J_1 - J_2 = (j_1 - j_2) p^{\sec z}.$$

The remaining terms are excluded. We can find the transmission index /217 p from the set of these equations. This method is very convenient, and can be employed reliably under the condition that the properties of the underlying surface are the same at different azimuths. Due to the southern location of Scutum, its azimuth changes within wide limits, between the culmination in the south, and the setting in the southwest. If the atmospheric transmission decreases as one moves to the west and as the observed object descends simultaneously, then the straight line which is determined by the coordinates $x = \sec z$ and $y = \lg(J_1 - J_2)$, is steeper with respect to the x -axes. This corresponds to a larger value of $\lg p$ and, consequently, to a smaller atmospheric transmission. Thus, the heterogeneity of the atmospheric optical properties with respect to the azimuth can lead to a fictitious value for its transmission. The same statement can be made regarding the determination of p with respect to the intensity of solar radiation at different zenith distances by employing the usual formula of Bouguer. Something similar can occur in the vicinity of Alma-Ata, where the south is covered by a double range of high snowy mountains above which the atmospheric transmission is undoubtedly higher than average. The entire northern half of the horizon is occupied by a dull desert which is covered by the usual haze. A change in the weather usually occurs due to the invasion of air masses from the northwestern corner of the horizon.

It can thus be assumed that the application of this method to the Milky Way, under the conditions of Alma-Ata, can produce a lowered value for the atmospheric transmission, which does not correspond to reality. It may be possibly better to determine p using photometric observations of stars, whose azimuth during the sunset or sunrise changes only within narrow limits - for example, α Aurigae or α Bootis.

However, let us determine p , by whatever method, within an

accuracy of approximately 1% for night observations. In this case, the equation

$$J - \text{red}(z) = j p^{\sec z} + CF(z)(p + 0.03)^{\sec z}$$

can include only the two unknown quantities j and C , which can be determined from the set of observations at different zenith distances. /218

The quantity j remains constant, and can be determined once and for all, if the observations are carried out in the same brightness units - for example, in the number of stars of the fifth magnitude per 1 square degree. In this case, by determining j for the days having particularly good atmospheric conditions, one can determine the unknown C for all the remaining observational days for which the quantity p cannot be determined with sufficient accuracy.

The magnitude of the atmospheric component around the celestial pole will be

$$CF(z_0)(p + 0.03)^{\sec z},$$

where

$$z_0 = 90^\circ - \varphi.$$

The observed brightness in this region J_3 depends on all three components of night airglow. As was indicated previously, the brightness of the galactic component outside of the atmosphere close to the pole is as follows, in the number of stars of the fifth magnitude per square degree:

$$j_0 = 0.399 \sim 0.4.$$

The observed galactic component is

$$j_0 p^{\sec z} + \text{red}(b, z_0).$$

The quantity $\text{red}(b, z_0)$, - i.e., the correction in the galactic component for the scattering of light in the troposphere - can be calculated according to the tables given above, in conformity with the value of p . For example, for $p = 0.85$ and $z_0 = 47^\circ$ (Alma-Ata), we have

$$j_0 p^{\sec z} + \text{red}(b, z_0) = 0.38.$$

It is interesting to note that the airglow close to Polaris, which is caused by the presence of the galactic component, barely depends on the atmospheric absorption, and can be assumed to be 0.37 - 0.38 in every case. Thus, for example, we have: /219

Transmission Index p	Apparent Galactic Component Close to the Celestial Pole $j_0 p^{\sec z} + \text{red}(b, r_0)$.
0.77	$0.272 + 0.086 = 0.36$
0.80	$0.289 + 0.080 = 0.37$
0.83	$0.304 + 0.071 = 0.37$
0.86	$0.321 + 0.061 = 0.38$

The brightness of zodiacal light around the celestial pole apparently is:

$$\text{Zodiacal light} = J_0 - CF(z_0)(p + 0.03)^{\sec z_0} - j_0 p^{\sec z_0} - \text{red}(b_0, z_0),$$

i.e.,

$$\text{Zodiacal light} = J_0 - CF(z_0)(p + 0.03)^{\sec z_0} - 0.37.$$

This conclusion is made upon the assumption that the portion of zodiacal airglow scattered into the lower atmospheric layers can be disregarded.

We should note that the determined magnitude of the zodiacal air glow represents the apparent zodiacal light around the celestial pole, i.e.,

$$\text{Zodiacal light} = (LZ_0) p^{\sec z_0} (1 + K),$$

where K, as previously used, represents the contribution of the troposphere component. We have compiled it in a table for different values of p (see Page 156). The first term $(LZ_0)p^{\sec z_0}$ is included in the over-all cosmic component j. The second term $(LZ_0)p^{\sec z_0}K$ represents the additional correction in the observed brightness J. Solving the equation

$$J - \text{red}(b_0, z) - (LZ) p^{\sec z} K = j p^{\sec z} + CF(z)(p + 0.03)^{\sec z},$$

when all the quantities in the lefthand side are known, we can determine the unknown j and C in the second approximation. This process of consecutive approximations can be continued until the new values of j and C do not differ from the preceding ones. /220

This general method for dividing the components is based upon the assumption that the ionosphere brightness distribution is described by the function $F(z)$, and does not depend on time throughout the given night. The latter assumption is far from being always valid. Let us assume that for each moment of time it is possible to

determine the total brightness of the night sky around the pole from observations. In addition, let the given night be characterized by constant transmission p . Consequently, a change depends exclusively on the ionosphere component around the pole, so that

$$J_0(t) - J_0(t_0) = \frac{(1+h)(p+0.03)^{\sec z_0} [C(t) - C(t_0)]}{\sqrt{(1+h)^2 - \sin^2 z_0}}.$$

At every other point in the sky, the change in this component with time is expressed by the formula

$$\frac{[J_0(t) - J_0(t_0)] F(z) (p+0.03)^{\sec z - \sec z_0}}{F(z_0)}.$$

If this quantity is subtracted from the value of $J(t)$, then the observations will be reduced to the moment t_0 , and the method presented above can be employed.

Thus, in the case of a variable ionosphere brightness, which is controlled at the celestial pole, the equations for dividing the components will be

$$\begin{aligned} J - \text{red}(b, z) - LZp^{\sec z} K - \\ - [J_0(t) - J_0(t_0)] \frac{F(z)}{F(z_0)} (p+0.03)^{\sec z - \sec z_0} = \\ = jp^{\sec z} + CF(z)(p+0.03)^{\sec z}. \end{aligned}$$

However, it must be noted that in practice reduction to one and the same moment t_0 can not always lead to the desired results.

Actually, this reduction assumes that the ionosphere brightness /221 distribution remains the same with zenith distance, and only the coefficient of proportionality C depends on time. This could hold only in the case of a rather low ionosphere height. In actuality, the height of the ionosphere airglow layers is on the order of 300 km. Therefore, the line of sight at a 75° angle to the normal intersects the luminescent layer at a point which is approximately 1200 km from the observer. Two sections of the layer which are visible at opposite azimuths close to the horizon are separated from each other by a distance of several thousand kilometers. If ionosphere airglow is caused by streams of charged particles, which produce a phenomenon which is similar to the aurora polaris, there is no basis for assuming that such streams will be completely uniform over a very large distance.

It is known that clouds of corpuscles, which produce magnetic storms, extend for a comparatively short distance. Direct observations on ionosphere airglow, which were carried out by Elvey at the MacDonald Observatory, showed that the brightness changed frequently with time in an entirely different manner in two regions of the sky, which were not

even particularly far from each other. I noted the same phenomenon at Alma-Ata when observing Scut. luminous, Scut. dark and Polaris. In several cases, a brightness change in the Polaris region did not correspond to a brightness change in the stellar cloud and the dark lane at Scutum - which, however, changed in the same manner.

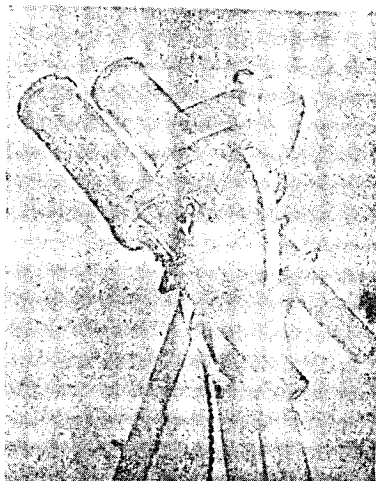
Due to this fact, it is preferable to perform observations on nights when the uniformity of the ionosphere airglow is not significantly disturbed, particularly when the brightness of the sky is as constant as possible around the celestial pole.

Using the above-described method, I observed Scut. luminous, Scut. dark and Polaris in the vicinity of Alma-Ata in the Fall months, when the Milky Way was in the most favorable position.

Observations were performed in 1945 at the Botanical Gardens of the Kazakh S.S.R. Academy of Sciences, in a location which was far removed from hills and was covered with thick vegetation. In 1946, observations were performed at a mountain astronomical station which was located 10 km south of Alma-Ata, which was much closer to the snowy peaks of the Zailiyskiy Altai. A photometer which I had constructed was used as the basic instrument. It had several interchangeable surfaces which were covered with a compound having constant luminescence and which was used as the light source.

/222

The compound having constant luminescence was zinc sulfide, which was activated with a radioactive substance in such a small amount that the brightness of the compound only slightly exceeded the densest star clouds of the Milky Way.



**Binocular Radioactive
Photometer**

As is known, the brightness of this compound is almost entirely independent of temperature, and also moisture (Ref. 74).

Therefore, the use of this compound has several advantages over the customary electric lamp, which is run on storage batteries and is controlled by an ammeter. One drawback of luminous compounds is their slow luminescence with the passage of time, due to which it is necessary to standardize them each week, and in any case each month. Standardization had to be employed also in our case, in order that all of the observed brightnesses be expressed in the number of stars of the fifth magnitude per square degree.

The light from the reference surface passed through the photometric wedge, which represented a gray, wedge-shaped band of colored gelatine held by Canada balsam between flat glass plates. /223

No matter how carefully it is constructed, such a wedge is not fully correct, and consequently cannot be characterized by a definite constant. Gelatine inevitably leaks onto its narrow edge, due to which the transmission of such a wedge - which changes, generally speaking,

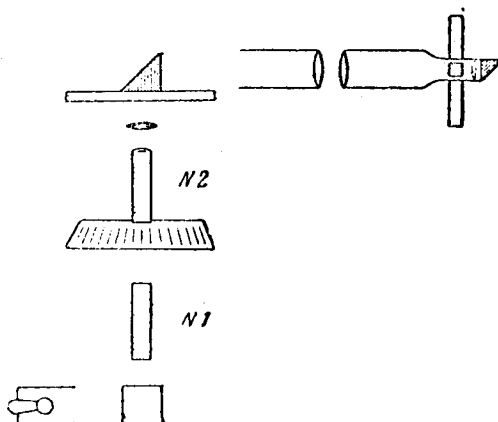


Figure 11
Arrangement for Studying Radioactive
Photometer Wedge

according to an exponential law - reaches a maximum close to the edge, and then again decreases. In any case, gelatine wedges must be studied throughout their entire length, especially under operational conditions. It is advantageous to use a Nagel' adaptometer for this purpose; this adaptometer is used to study the sensitivity threshold of the retina. There is a rather large reference surface in this instrument, whose brightness changes within wide limits with the aid of a "cat's eye" and several auxilliary devices. I employed other methods to study the scale of a photometric wedge. The most convenient method, which I employed for the greatest number of measurements, is as follows. em-/224

The light from a small electric lamp, which runs on storage

batteries and is controlled by a potentiometer, falls on the silver-plated surface of a Lummer prism. This light is reflected in the form of a thin bundle at a rightangle to the optical axis of two nicols placed one behind the other. The first nicol N_1 is fixed; the second N_2 nicol is connected with a graduated circle, which makes it possible to determine the angle between the polarization planes of both nicols. The upper bundle passes through a strong converging lens and a green Schott filter made of ground glass. This glass is mounted in a horizontal direction. A small prism having complete internal reflection is placed upon it; this prism deflects the light into the collimator. A parallel bundle of light passes out through the collimator lens; this bundle completely covers the photometer lens with the wedge which is being studied. The brightness of the incoming bundle can be changed in a definite manner by turning the nicol. The observations are carried out at different wedge settings, which can thus be reduced in accordance with the polarization scale throughout its length. In order to increase the aiming accuracy, the brightest luminescent surface is used, which is 11.7 times brighter than normal, but of the same clear green color. The color is absolutely the same as the background of the sky during nocturnal observations, when the normal reference surface is employed. When the wedge scale was being studied, it was necessary to compensate for the color difference with the small lamp, using a green filter; this was necessary due to the significant increase in the brightness of the field of vision. Figure 11 shows a diagram of this arrangement.

The wedge was studied by four different persons, whose eyes were adapted beforehand to total darkness. It was found that the manner in which the brightness changed with the wedge can be represented by the usual formula, within the limits of the scalar readings from 26 to 80: 225

$$\lg J = C - KS,$$

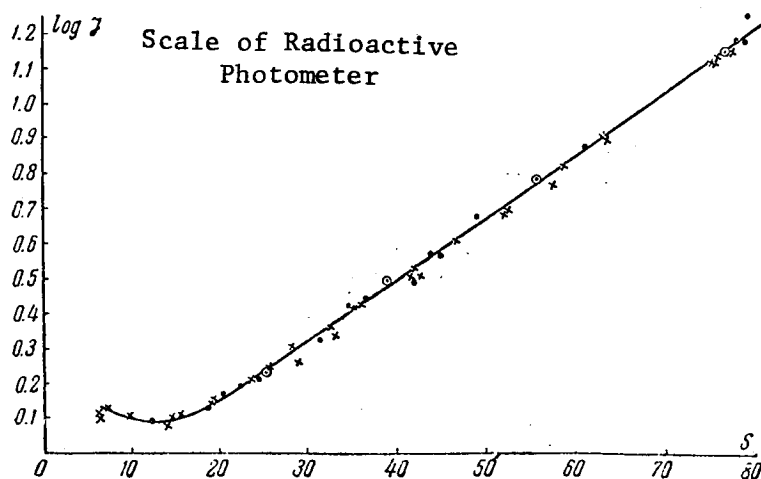
where the wedge constant K equals 0.0175, if S is expressed in millimeters. This corresponds to 0.438 stellar magnitudes per 1 cm.

Within the limits from $S = 6$ to 26 mm, the brightness curve followed another course, and was represented by an additional empirical table. Figure 12 presents the results derived from studying the wedge (the scale readings S are plotted along the x-axes; $\lg J$ - along the ordinate axis).

Let us describe the optical characteristics of a radioactive photometer designed for visual determination of the nocturnal sky brightness (Figure 13). The lens is simple eyeglass, and has a diameter of 40.6 mm and a focal distance of 215 mm. The ocular, which is an achromatic lens which has been cemented together, has a focal distance of 37 mm and a diameter of 18 mm. The ocular aperture has a diameter of 8mm, which exceeds the pupil of the human eye even when completely

adapted to the dark. In this way, during observations of the sky

/226



/225

Figure 12

Scale of Radioactive Photometer

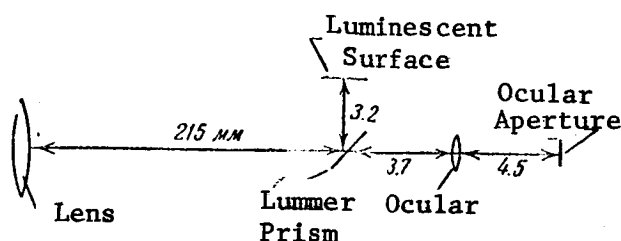
background or of star clouds in the Milky Way, there is no loss of light in the optics, with the exception of unavoidable absorption in its interior and reflection from the lens surface, which does not exceed 20% in all. The ocular lens is focused on the silver-plated surface of the Lummer prism. This surface represents the projection of a square, whose side equals 2 mm.

/226

The surface of constant luminescence is located in a small side tube at a distance of 32 mm from the optical axis of the photometer, and is bounded by a diaphragm which is about 8 mm in diameter. The light from it, after being reflected from the silver-plated surface of the Lummer prism, passes into the ocular, and forms the outgoing bundle having a diameter which is necessarily larger than 8 mm, i.e., which completely covers the pupil of the observer's eye. The eye is placed on the ocular pupil at a distance of 45 mm from the ocular lens. The photometric wedge in its mounting is located almost in the immediate vicinity of the Lummer prism, to the extent that this is technically possible. The photometric wedge has a simple recording device which makes it possible to plot the readings on the paper band placed upon it. The observational journal is used to record only the objects observed and the corresponding times; the readings of the recording tapes are usually made on the following day. This enables the observer to be completely objective.

The photometer is mounted on an azimuthal stand. The circle is divided every 10°, and the dividing lines are lined with a luminescent substance which is clearly visible in the dark. Readings are made

/227



/226

Figure 13

Diagram of Radioactive Photometer.

up to 1° . One important problem is the standardization of the device,^{/227} which - as was indicated previously - must be repeated at least once a month. Due to the significant difficulty entailed in standardizing such faint brightness, the decision was reached to perform the measurements required for this purpose every time that it was possible.

Different standardization methods can be employed, which we shall briefly describe below. The most direct method consists of shifting the lens from the focus so as to obtain an extrafocal image of the star with a surface of several square degrees. For stars of the 2 - 3rd magnitude, this image exceeds the sky background brightness by approximately a factor of 10, and can be readily measured photometrically employing the brightest luminescent surface. The brightness of the adjacent sky background is determined with the aid of a regular luminescent effective area. In order to determine the diameter of the extrafocal image, the time required for it to pass through a fixed point in the field of vision is recorded, using a stationary instrument; the angle of the reference area in the Lummer prism is used as the fixed point. For this purpose, the brightest stars must be employed - for example, α Aquilae - which are located as close as possible to the meridian.

Let j_1 be the brightness of the extrafocal image, i.e., star plus background, and j_2 the brightness of the adjacent sky background. Let the stellar magnitude of the star which is chosen for the standardization be m . Finally, let p be the transmission index of the atmosphere at the moment of observation; z - the zenith distance of the star; ω - the area of its extrafocal image expressed in square degrees. The ratio of the star brightness to the background is

$$\frac{\text{star}}{\text{background}} = \frac{j_1}{j_2} - 1 \quad (20)$$

The brightness of the extrafocal image of the star - as it would appear through an absorbing atmosphere in the absence of a superposed

sky background - in the number of stars of the fifth magnitude per 1 square degree is

/228

$$\text{star} = \frac{2.512^{5-m} p^{\sec z}}{\omega}.$$

Consequently, the brightness of the sky background, corresponding to the readings ϕ on the photometer scale, will be as follows on the basis of formula (20):

$$\text{background} = \frac{2.512^{5-m} p^{\sec z} j_2}{\omega(j_1 - j_2)}.$$

This determines the brightness, expressed in absolute stellar units, which corresponds to a specific reading on the photometric wedge, which solves the problem which has been formulated.

This method is fairly accurate and can be readily employed, but it assumes that the extrafocal image is completely uniform. Because this is not the case, it is necessary to study the brightness distribution throughout the extrafocal image. Let this distribution be represented by the function $f(r)$. The preceding considerations refer to a uniform disc, i.e., to its mean brightness j_m

$$j_m = \frac{k \int f(r) r dr}{\int r dr},$$

k represents the proportionality coefficient.

If the observed brightness of the extrafocal image is j_r at the distance r from its center, we then have

$$j_m = j_r \frac{\int f(r) r dr}{\int f(r) r dr}.$$

In our case, reduction to a uniform disc comprises $\Delta s = + 7.2$ mm in the photometer scale for a photometric measurement of the image center, and only $\Delta s = + 1.8$ mm for a photometric measurement of its mean section for $r = 0.5$ - as was usually the case in our observations. /229
The significant non-uniformity of the extrafocal image is caused by the fact that the objective consists of a simple lens in order to avoid unnecessary losses of light.

Another standardization method, which is quite convenient to use with photometers of any construction, employs a luminescent screen which covers the floor of a cylindrical box.

The brightness of this screen can be measured with a photometer, but it may be necessary to attenuate it first with a rotating disc having a certain sectorial slit. If this box is covered with a lid

having a small opening, on the order of 2 mm in diameter, it is possible to obtain almost a point light source - an artificial star - whose brightness will change in a manner which is inversely proportional to the distance to the eye of the observer.

At the normal brightness of luminous compounds, it is possible to have the brightness of the artificial star equal the reference star of the second or third stellar magnitude when the box is at a distance of 1 - 2 m.

The greenish color of the artificial star is barely discernible, and does not prevent making valid comparisons with stars of type A. One great disadvantage is represented by the fact that the luminescent opening is outlined on the dark background of the lid, in contrast to a natural star on which the rather light background of the night sky is superimposed. In order to eliminate this difficulty, the lid must be covered on the outside with a piece of white paper, and if necessary a foreign light source must be used for a short period of illumination, so that the lid itself almost merges with the background of the night sky in a given direction.

Let L be the brightness of the luminescent compound which covers the floor of the box, i.e., the stream of radiation which is emitted by a unit of surface per unit of solid angle; ρ the opening radius in the floor of the box; l - its distance from the observer, at which the brightness of the artificial star appears the same as that of the reference star having the magnitude m at the zenith distance z . It is necessary to express the brightness L in the number of stars of the /230 fifth magnitude per 1 square degree.

The brightness L is equal numerically to the illumination produced on a surface placed in a normal direction by an area of the screen having dimensions of 1 steradian. At the distance l , the opening in the lid produces the illumination

$$L \frac{\pi \rho^2}{l^2}$$

In its turn, a star of the m^{th} magnitude produces an illumination, in the units given above, which equals

$$2.512^{5-m} p^{\sec z},$$

where p is the atmospheric transmission index (it is assumed that the effect of the background is excluded).

Under observational conditions,

$$\frac{L \pi \rho^2}{l^2} = 2.512^{5-m} p^{\sec z},$$

from which L is found in the number of stars of the fifth magnitude per one steradian.

Performing a calculation per square degree, we shall have

$$L = \frac{2.512^{5-m} \rho^{\sec z}}{\pi \rho^2} l^2 \left(\frac{\pi}{180} \right)^2.$$

If, for example, $\rho = 1 \text{ mm}$ and $l = 1000 \text{ mm}$, then we have the following when the brightness equals a star of the third magnitude:

$$L = \frac{6 \cdot 10^6}{3} \cdot \frac{0.6}{3600} = 400.$$

Under these conditions, the screen brightness is approximately one hundred times greater than the night sky, and therefore - in order to facilitate observations with a radioactive photometer - it is necessary to attenuate it with the aid of a rotating disc by approximately 10 - 15 times.

From these observations, we can express the scale readings S in the number of stars of the fifth magnitude per square degree, with which the necessary standardization is achieved. To the best of my knowledge, this method was first employed by the student Kapteyna Intema (Ref. 75) in 1908 when studying the brightness of the night sky.

/231

The third standardization method is even simpler, and necessitates observations without using an objective. Instead of the objective, a diaphragm is used which outlines a certain solid angle ω of the celestial vault. A star having a known brightness is observed through the weak ocular of a photometer, and a photometric measurement is made of the circle of confusion of the stellar image, as compared with the surrounding sky background. In this case, we find complete similarity with the first standardization method, while the calculations can be carried out according to a formula of the same form

$$\frac{2.512^{5-m} \rho^{\sec z}}{\omega \cdot \text{background}} = \frac{j_1}{j_2} - 1.$$

However, it can be shown that this method is extremely disadvantageous when applied to stars, since the contrast between the circle of confusion and the retina and the over-all sky background will be negligible, even in the case of bright stars.

Let us determine the radius for the circle of confusion. Let the

ocular just touch the pupil of the eye. If F_1 is the focal distance of the ocular and F_2 is the pupil, then the focal distance of the conjugate lens - ocular + pupil will be

$$F = \frac{F_1 F_2}{F_1 + F_2}.$$

If we use d to designate the radius of the pupil and if it is assumed that the distance from the retina to the pupil equals its focal distance F_2 , we find the following for the radius ρ of the circle of confusion:

$$\rho = d \frac{F_2 - F}{F}$$

or

$$\rho = d \frac{F_2}{F_1}.$$

/232

Consequently, the angular surface of the circle of confusion will be

$$\omega_1 = \frac{\pi \rho^2}{F_2^2} = \frac{\pi d^2}{F_1^2}.$$

The brightness of the circle of confusion equals the total amount of light entering the pupil from a star and divided by the angular surface ω , i.e.,

$$L \pi d^2 : \frac{\pi d^2}{F_1^2} = L F_1^2,$$

where L represents the illumination from the observed star having the m th magnitude.

It can be readily seen that this brightness does not change no matter what the position of the eye with respect to the ocular. Thus, for example, if the pupil is located at the main focal point of the ocular, the total amount of light entering from the star will be

$$L \pi R^2 \text{ (R - ocular radius).}$$

The angular dimensions of the ocular lens, for a given position of the eye, will be

$$\Omega = \frac{\pi R^2}{F_1^2}$$

and therefore the desired brightness will be, just as previously,

$$\frac{L \pi R^2}{\Omega} = L F_1^2.$$

Let us determine the brightness of the background. The diaphragm outlines a section of the solid angle

$$\omega = \frac{\pi r^2}{l^2} \quad (r - \text{diaphragm radius, } l - \text{tube length}).$$

If L_ϕ is the illumination from a unit of solid angle in the sky, then the ocular receives the amount of light

$$\omega L_\phi \pi R^2.$$

All of this light penetrates the pupil of the eye, and causes uniform illumination of the ocular, whose solid angle is

/233

$$\frac{\pi R^2}{F_1^2}.$$

Consequently, the brightness of the background in this case ($\rho < d$) is

$$L_\phi \frac{\pi r^2}{l^2} \pi R^2 : \frac{\pi R^2}{F_1^2}$$

or

$$L_\phi \pi \rho_1^2.$$

In this case, the star/background ratio will be

$$\frac{L}{L_\phi} \frac{\rho}{\pi r^2} \quad \rho_1 < d.$$

If, on the other hand, $\rho_1 \geq d$, then this ratio will be

$$\frac{L}{L_\phi} \cdot \frac{F_1^2}{\pi d^2}.$$

It can be readily seen that this method is not very practical, even in the case of very weak oculars. Thus, for example, in our case

$$F_1 = 3.5 \text{ cm, } d = 0.4 \text{ cm, } \frac{F_1^2}{\pi d^2} \approx 20.$$

The brightness of the night sky approximately equals 2 - 3 stars of the fifth magnitude per 1 square degree, or approximately 10,000 per steradian. Therefore, even for stars of 0 magnitude,

$$\frac{L}{L_\phi} \approx 0.01.$$

and therefore

$$\frac{\text{Stellar brightness outside of atmosphere}}{\text{brightness of sky background}} \approx 0.2.$$

Thus, even in the case of very bright stars, contrast between their image and the surrounding background is negligible. Exact measurements are not possible. /234

In the case of a photometer which is designed as described above, such a method is not applicable for fundamental reasons. Actually, this method entails the significant assumption that all of the light from the sky background, which is outlined by the diaphragm, enters the pupil ($\rho < d$), as occurs with the light from the reference star.

If this is not the case - i.e., if $\rho > d$ - then the brightness of the sky background will of necessity depend on the size of the pupil, and the brightness of the stellar image will be independent of it. Therefore, the variability of the pupil is not excluded, and photometric measurements are impossible.

If the condition $\rho < d$ holds, and is not restricted by the loss of light, then this will not correspond to our photometer design, in which the exit pupils of both the objective and of the reference source considerably exceed the pupil of the eye dimensions.

The last standardization method of our photometer was employed by A. A. Kalinyak, but it did not lead to any results for the reason given above.

I prefer to use the first standardization method - the method of extrafocal images. As a result, it was possible to derive an absolute scale of the radioactive photometer for each observational period. Thus, for example, in October, 1945, the brightness in 2.14 of our units corresponded to a scale reading of $S = 40$; and at the beginning of July, 1946 - 1.74. This change is very small. It must be noted, however, that the area with the continuously-active compound, which was used in the photometer, was stored for several years.

Let us now turn to the problem of dividing the components of night sky airglow. The observations in October, 1945, were performed during completely clear weather, but after a long period of drought - when the lower layers of the earth's atmosphere produced enormous solar coronas and the sun, approaching the horizon, became almost invisible. Apparently, the atmospheric transmission, which could not be great during the /235 day, constantly increased at night. This could be determined, for example, by the rising of the crescent moon in the pre-dawn hours. First, the dark portion of the lunar disc appeared above the mountain tops, slightly illuminated by earthlight, and then the bright crescent appeared. This indicated great transmission of the air above the mountain tops, and an over-all improvement of the atmospheric conditions as compared with diurnal conditions.

We should point out that large solar coronas do not necessarily indicate poor transmission, which depends on the over-all size of the

3, Oct., 1945

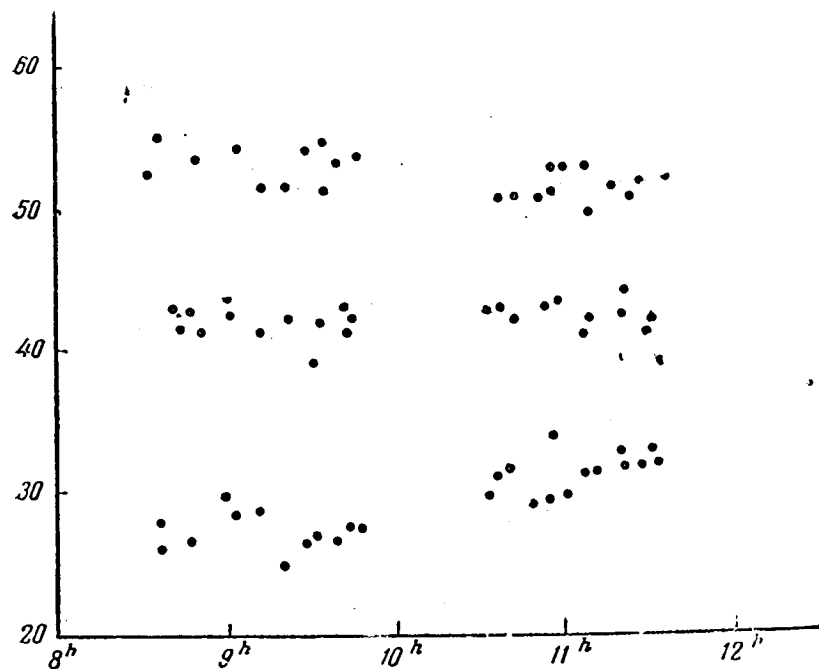


Figure 14

Observations on October 3, 1945.

scattering indicatrix. One characteristic property of Central Asia /236 is good visibility away from the sun, but poor visibility toward the sun.

We can find the transmission index p primarily from observations of Scut. luminous, Scut. dark and Polaris.

Figure 14 presents observations which were carried out on October 3, 1945.

The working hours are plotted along the abscissa axis (their correction was 8^m45^s), and the scale readings of the photometer in millimeters are plotted along the ordinate axis. The Scut. luminous region represented the brightest and most uniform stellar field ($\alpha = 18^h 50^m$, $\delta = -7^\circ$), which could be readily found by the configuration of telescopic stars.

Scut. dark ($\alpha = 18^h 40^m$, $\delta = -6^\circ$) was observed in the darkest portion of the lane to the west of the first object.

The following system of equations is given below:

$$\lg(j_1 - j_2) + \sec z \lg p = \lg(J_1 - J_2).$$

The coefficients were read off along the averaged curves every 30m.

$\lg(j_1-j_2)+1.662$	$\lg p=0.2304$
1.788	0.1987
1.982	0.1847
2.265	0.1644
2.693	0.1038
3.440	0.0414
4.856	-0.0506

We thus have the normal equations

$$\begin{aligned} 7 \lg(j_1-j_2)+18.686 \lg p &= 0.8728 \\ 18.686 \lg(j_1-j_2)+57.639 \lg p &= 1.6529 \end{aligned}$$

Their solution yields

$$j_1-j_2=0.356; \lg p=-0.0866$$

and, consequently,

$$p=0.82.$$

The solution is primarily determined by points which are close to the horizon, where there can always be some doubt with respect to the uniformity of the atmosphere. Discarding the last conditional equation, we find

/237

$$p=0.78.$$

If, on the other hand, we discard the first equation, retaining all of the others, we then have

$$p=0.824.$$

Performing a graphic solution, in which the relative importance of the individual observations is distributed in the best manner, we find

$$p=0.813.$$

We can finally assume that $p = 0.82$. If this value for the atmospheric transmission accurately satisfies the conditional equations, then observations on either the light stellar cloud, or on the dark lane, can be equally employed when dividing the components. The conditional equations for dividing the components have the following form:

j	$0.720+C$	$1.176=1.89$	j	$0.587+C$	$1.383=1.93$
	0.702	1.218=1.89		0.506	1.398=1.89
	0.675	1.274=1.94		0.382	1.253=1.96
	0.639	1.333=1.93			

It was thus assumed that $h = 0.05$. The atmospheric component is represented by the usual expression

$$\frac{C(1+h)(p+0.03)^{\sec z}}{\sqrt{(1+h)^2 - \sin^2 z}}.$$

The small change in the sky brightness around the pole was not taken into account. On the average, the solution yields

$$C = 0.95,$$

from which it follows that the atmospheric component for the pole ($z = 47^\circ$) will be 1.045.

Since, in reality, the observed brightness around the pole is about 1.30, and the stellar component in the same region is 0.37, we can see that nothing remains for the portion of zodiacal light.

/238

There is no doubt that the transmission index along the Milky Way was too small for the nights of October 6 and 7 - namely, 0.79 and 0.74. This leads to an excessively large value for the atmospheric component around the pole, which exceeds the entire observed quantity. If an increased transmission value of $p = 0.82$ is used for these days, which is identical with that of October 3, then we obtain much more reasonable results. However, in this case there is no zodiacal light around the celestial pole. The results can be improved with a more rigorous treatment - for example, by taking galactic diffused light into account. However, there is no doubt that their nature remains unchanged.

Observations in 1945 were not entirely reliable, due to mediocre atmospheric conditions and due to the fact that the transmission probably varied during the night.

In 1946, the atmospheric conditions were incomparably better. There was rain for a large portion of the year, with small intervals of dry, clear weather. In addition, the observations were performed in the mountainous area of the Institute of Astronomy and Physics of the Kazakh SSR Academy of Sciences, at an altitude of 1350 m - i.e., at approximately 600 m above the observational area in 1945. Observations on the same regions of the Milky Way - Scut. luminous, Scut. dark and the region around the celestial pole - were carried out in August, 1946, usually beginning with the most southerly position of Scutum and proceeding almost up to its setting. It was usually found that the brightness of the stellar cloud slowly increased as the zenith distance increased, or remained approximately constant, reached a certain maximum, after which it began to rapidly decrease. For the dark lane of Scut. dark, this brightness increase with zenith distance was always expressed very clearly. This indicates the existence of a luminescent atmospheric veil, whose intensity follows the formula $CF(z)(p+0.03)^{\sec z}$, according to our theory.

/239

If both areas - bright and dark - were observed simultaneously throughout the entire night at the same zenith distance, then the atmospheric airglow would be entirely eliminated when comparing the difference $J_1 - J_2$, independently of its fluctuations with time. In our case, both areas were observed at different z , due to an increase in the parallel inclination. We can derive the mean curves on the basis of all the observations, and can record the photometer readings S_1, S_2 which pertain to the same zenith distance, but to somewhat different moments in time. This fact, however, is not of great importance.

The determination of p , which is carried out according to the formula

$$J_1 - J_2 = (j_1 - j_2) p^{\sec z},$$

is given in the following Table:

1946	p	Comments
7 - 8 August	0.78	August 8, morning, completely clear. Cloudy horizon in west & north.
23 - 24 August	0.82	
24 - 25 August	0.74	
25 - 26 August	0.81	
31, Aug. - 1, September	0.81	Clear; hazy zone in north.
1 - 2 September	0.84	

These results were obtained graphically by drawing lines which best represented the coordinates $y = \lg(J_1 - J_2)$ and $x = \sec z$, or more precisely the atmospheric masses m_z . These atmospheric masses, which represented an increase in optical thickness with zenith distance, can be calculated with a fair amount of accuracy by means of the series:

$$m_z = \sec z (1 + h_0) - \frac{1}{2} \sec^3 z (2h_0 + 6h_0^2 - 6h_0^3) + \frac{3}{8} \sec^5 z (8h_0^2 + 48h_0^3 + 120h_0^4 + 120h_0^5) + \dots, \quad /240$$

where h_0 designates the height of the uniform atmosphere; it is assumed that the radius of the terrestrial sphere equals unity. In the case of a purely gas atmosphere, whose structure is determined by a barometric formula, h_0 is very close to 8 km. The uniform height of a real atmosphere, whose absorption is determined to a significant extent by aerosols also, must be smaller, since aerosols are connected with lower layers. If $h_0 = 0$, then we have

$$m_z = \sec z,$$

and this must be a good appropriation in the general case, up to $z = 75^\circ$.

The values of p , determined for the Milky Way, cannot be regarded as real. There is no doubt that the large atmospheric transmission to the south, over snowy mountain peaks, causes a fictitious decreased value for the transmission index.

Let the transmission index p_1 correspond to the zenith distance z_1 , and the quantity p_2 correspond to z_2 . As a result of a change in the azimuth, we find that

$$z_1 < z_2, \text{ but } p_1 > p_2.$$

Let the observed brightnesses of a star be J_1 and J_2 , so that

$$J_1 = J_0 p_1^{\sec z_1}; J_2 = J_0 p_2^{\sec z_2},$$

where J_0 is the brightness of the star at the boundary of the atmosphere.

Employing the customary computational procedure, we find p according to the formula

$$\lg p = \frac{\lg J_1 - \lg J_2}{\sec z_1 - \sec z_2} = \frac{\sec z_1 \lg p_1 - \sec z_2 \lg p_2}{\sec z_1 - \sec z_2}$$

or

$$\lg p = \lg p_1 - (\lg p_1 - \lg p_2) \frac{\sec z_2}{\sec z_2 - \sec z_1}. \quad (21)$$

If, for example, for $\sec z_1 = 1$, $p_1 = 0.85$ and for $\sec z_2 = 4$, /241
 $p_2 = 0.84$, then - assuming that p is constant, as is usually done when analyzing observations - we find the following according to (21):

$$p = 0.83,$$

i.e., a fictitiously decreased value.

Thus, the decrease in the atmospheric transmission toward the west, where the effect of the desert comes into operation, can actually be of importance. In addition to this, the presence of local haze in the west, which is frequently observed over deserts, can have an effect also.

A criterion can be formulated for the validity of these conclusions. If the transmission index is fictitiously small, then the difference $(J_1 - J_2)_0$ outside of the earth's atmosphere must be too large. In actuality, we have:

		$J_1 - J_2$	p	$(J_1 - J_9)^*_0$
7 - 8 August	$z = 70^\circ$	1.49	0.78	3.06
23 - 24 August	"	1.27	0.82	2.26
24 - 25 August	"	1.20	0.74	2.88
25 - 26 August	"	1.55	0.81	2.86
31, Aug. - 1 September	"	1.46	0.81	2.69
1 - 2 September	"	1.44	0.84	2.38

It can thus be seen that the smallest value $(J_1 - J_2)_0$ outside of the atmosphere was obtained on the clearest night on September 1 - 2, while on nights which were not clear ($p = 0.74$ and 0.78) the largest values were obtained.

Due to this fact, we shall now use somewhat larger values of atmospheric transmission on the order of $0.85 - 0.86$. This is in agreement with the determinations of p using a halo photometer which I constructed; these determinations were made in the same area in the early morning hours, beginning with sunrise. Thus, much larger transmissions were /244 invariably obtained, on the order of 0.85 , which are characteristic in general for this locality for visual rays.

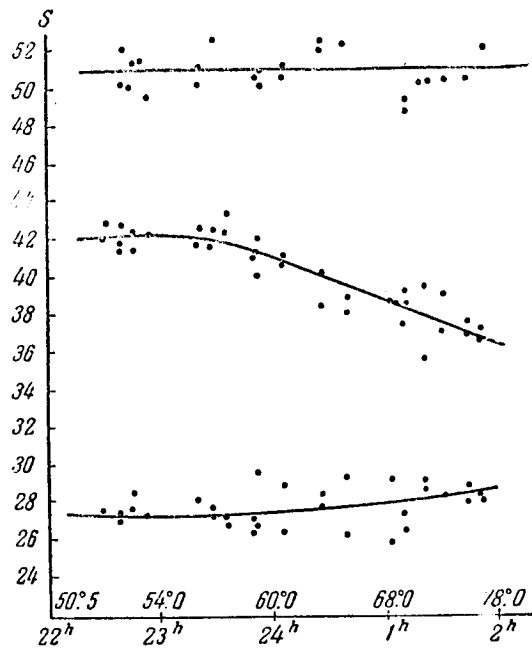
Table 33 presents observational data on the days indicated, derived from smoothed-out curves. The individual observations are presented in the adjoining graph.

We should note that the readings of the photometer scale S_1, S_2, S_3 , were usually made in rapid succession. The corresponding brightnesses J_1, J_2, J_3 are expressed in the number of stars of the fifth magnitude per square degree. A reduction for the galactic light, scattered in the troposphere, is given in the last column. /245 This correction is on the order of 0.1 in our units; it changes very little with zenith distance, and is of no significant importance. It can be demonstrated that reduction for zodiacal light, which is scattered in the troposphere, can be completely disregarded without any effect.

Let us first examine the observations on August 25 - 26, 1946, which were performed under very good conditions. Judging by the uniformity of J_3 , we can assume that the state of the ionosphere was stable. Introducing a correction for scattered galactic light, we can find the following system of conditional equations for $p = 0.81$, determined for the Milky Way (based on the determinations of Dewar and Karimov, it is assumed that the height of the ionosphere layer

* Note: Original foreign text gives J_9 ; correct term should be J_2 .

August 23 - 24, 1946



/242

August 24 - 25, 1946

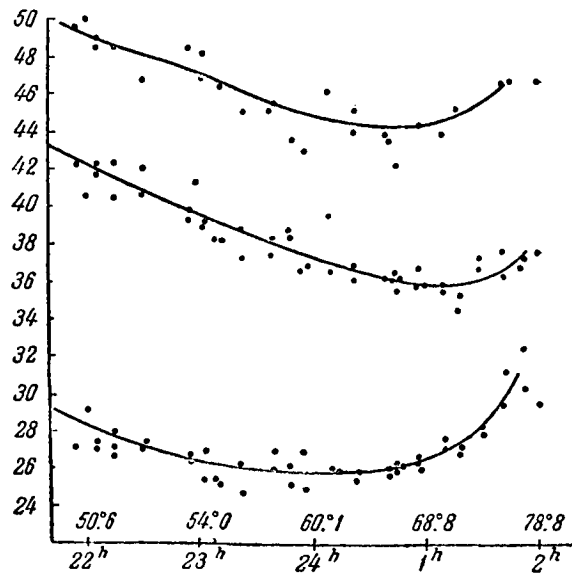


Figure 15

Observations on August 23 - 24 and 24 - 25, 1946.

is 0.046):

/245

$z=50^\circ$	C	$1.123+j$	$0.0721=4.19$
55		1.192	$0.693=4.23$
60		1.266	$0.656=4.28$
65		1.336	$0.608=4.24$
70		1.386	$0.540=4.12$
72		1.333	$0.506=4.04$
74		1.370	$0.466=3.97$

TABLE 33

/243

Scut. lumin.	Scut. dark	z	sec z	Pol	J_1	J_2	J_3	red	
S_1	S_2	S_3 7-8 Aug. 1946							
25.0	35.8	70°	2.92	49.6	4.22	2.73	1.58	0.13	$p=0.78$
25.7	35.8	72	3.24	48.7	4.11	2.73	1.62	0.15	
26.8	35.8	74	3.63	48.4	3.93	2.73	1.64	0.16	
27.8	35.9	76	4.13	48.3	3.78	2.71	1.65	0.17	
29.0	36.2	78	4.81	48.5	3.60	2.68	1.63	0.18	
30.4	36.4	80	5.76	48.8	3.40	2.66	1.61	0.20	
23-24 Aug. 1946									
27.5	42.3	50°	1.56	51.0	3.82	2.10	1.47	0.07	$p=0.8$
27.6	42.0	55	1.74	50.9	3.81	2.12	1.43	0.08	
27.8	41.2	60	2.00	50.8	3.78	2.20	1.48	0.09	
28.0	39.7	65	2.37	50.8	3.75	2.34	1.48	0.10	
28.3	38.7	70	2.92	50.7	3.71	2.44	1.49	0.12	
28.4	38.2	72	3.24	50.7	3.69	2.48	1.49	0.13	
28.5	37.8	74	3.63	50.7	3.68	2.52	1.49	0.14	
24-25 Aug. 1946									
28.0	43.2	50°	1.56	49.3	3.75	2.03	1.58	0.09	$p=0.81$
26.2	39.0	55	1.74	47.0	4.03	2.41	1.74	0.10	
25.8	37.4	60	2.00	45.0	4.09	2.56	1.89	0.11	
26.0	36.3	65	2.37	44.2	4.06	2.67	1.95	0.13	
27.0	36.0	70	2.92	44.7	3.90	2.70	1.91	0.15	
27.8	36.0	72	3.24	45.4	3.78	2.70	1.86	0.16	
29.0	36.1	74	3.63	46.0	3.60	2.69	1.81	0.17	
25-26 Aug. 1946									
24.7	40.5	50°	1.56	46.0	4.27	2.26	1.81	0.08	$p=0.81$
24.4	39.4	55	1.74	44.8	4.32	2.37	1.91	0.09	
24.0	38.1	60	2.00	44.7	4.38	2.49	1.91	0.10	
24.2	37.0	65	2.37	44.8	4.35	2.60	1.91	0.11	
24.8	36.0	70	2.92	45.0	4.25	2.70	1.89	0.13	
25.3	35.7	72	3.24	45.0	4.17	2.74	1.89	0.13	
25.7	35.4	74	3.63	44.8	4.11	2.78	1.91	0.14	

(continued on next page)

Table 33 (continuation)

Scut. lumin.	Scut. dark	z	$\sec z$	Pol	J_1	J_2	J_3	red	
S_3 31 Aug. - 1 Sept. 1946									
26.2	39.4	65	2.37	46.7	4.03	2.37	1.76	0.11	$p=0.81$
26.8	38.3	70	2.92	47.4	3.93	2.47	1.71	0.13	
27.2	37.9	72	3.24	47.7	3.87	2.51	1.69	0.13	
27.7	37.7	74	3.63	47.8	3.79	2.53	1.68	0.14	
28.6	37.9	76	4.13	47.7	3.66	2.51	1.69	0.16	
29.0	38.4	78	4.81	47.6	3.45	2.46	1.70	0.17	
30.0	39.0	80	5.76	47.2	3.18	2.41	1.73	0.18	
1-2 Sept. 1946									
27.5	40.5	60	2.00	9.4	3.82	2.26	1.58	0.09	$p=0.84$
27.1	40.4	65	2.37	50.0	3.89	2.27	1.54	0.10	
27.5	39.3	70	2.92	50.0	3.82	2.38	1.54	0.12	
27.8	39.1	72	3.24	50.0	3.78	2.40	1.54	0.13	
28.3	38.9	74	3.63	50.0	3.71	2.42	1.54	0.14	
29.0	38.9	76	4.13	50.0	3.60	2.42	1.54	0.15	
30.0	39.0	78	4.81	50.0	3.45	2.41	1.54	0.16	

A solution of these equations which employs the Cauchy method yields the following:

$$C = 1.85; j = 3.00.$$

We thus find that the ionosphere component around Polaris is

$$1.85 \cdot 1.084 = 2.01.$$

The total brightness of the sky observed in the same region is only 1.91 in all. Thus, nothing remains for the portion of galactic and zodiacal components. This again demonstrates the illusory nature of the value obtained for p . Actually, if these calculations are repeated for successive p of different magnitudes, we then obtain the following:

TABLE 34

25 - 26 AUGUST, 1946

p		C	Reduction for pole	Galactic component around the pole	Observed bright- ness at the pole	Zodiacal component
0.81	3.0	1.85	1.084	0.37	1.91	-0.47
0.82	3.2	1.635	1.104	0.37	1.91	-0.26
0.83	3.3	1.49	1.126	0.37	1.91	-0.14
0.84	3.4	1.36	1.145	0.38	1.91	-0.03
0.85	3.6	1.19	1.167	0.38	1.91	0.14

It can thus be seen that the results depend to a significant extent upon the assumed value of atmospheric transmission. The atmospheric component of the night sky luminosity decreases with an increase in p . A plausible value for C is obtained only for the maximum assumed value of p , which is generally characteristic for the point of observation. However, in the latter case the zodiacal component around the pole is very small, if it occurs at all.

Let us examine several other days. On the night of August 23, 24, the brightness around Polaris was almost constant. Assuming that $p = 0.84$ and 0.85 , we can calculate the following values for the components:

TABLE 35

p	C	j	Ionosphere component at the pole	Galactic component at the pole	Observed brightness at the pole	Zodiacal component at the pole
0.84	1.10	3.2	1.26	0.38	1.48	- 0.16
0.85	0.97	3.3	1.13	0.38	1.48	- 0.03

Thus, the zodiacal component at the pole can be positive, although it may be a very small quantity, only for a value of p which exceeds 0.85.

Observations on August 24 - 25, encountered special difficulties, /247 since the brightness of the sky around the pole regularly increased during the first half of the night. At sunrise, extensive light cirro-stratus clouds appeared around the northern and eastern horizon. If a correction is introduced for the ionosphere brightness change according to the formula given above and if it is assumed that $p=0.85$, we then find

$$C = 1.02; \quad j = 3.7; \quad \text{mean } J_3 = 1.90.$$

The zodiacal component at the pole is 0.15.

If a reduction is not made for the ionosphere brightness change, then for $p = 0.85$ the zodiacal component increases considerably.

On the other hand, for $p = 0.74$ which is determined for the Milky Way, this component assumes a large negative value which is impossible.

Observations on September 1 - 2 were performed under very good atmospheric conditions, but were initiated comparatively later due to the presence of a young moon. The brightness around the pole remained approximately constant. For $p = 0.85$, we find

$$C = 1.167; j = 3.0.$$

For the ionosphere component around the pole, we find

$$1.16 \cdot 1.167 = 1.354.$$

The over-all observed brightness in this region is 1.54.

Consequently, the zodiacal component is 0.19. If it is assumed that $p = 0.87$ according to the determinations of Ye. V. Fesekov (halo photometer), we find

$$C = 0.93; j = 3.20.$$

For the pole, we have the ionosphere airglow

$$0.93 \cdot 1.207 = 1.123,$$

from which a zodiacal component is found of 0.04.

Observations on 31, August - 1, September, 1946 were initiated later due to the presence of the moon. The sky background around Polaris changed with time, but not to the extent that it did on August 24 - 25. If a mean value of $p = 0.85$ is assumed, we then find /248

$$j = 3.3; C = 1.11.$$

The observed brightness around Polaris is 1.69, and the ionosphere component is

$$1.11 \cdot 1.167 = 1.30.$$

Adding the galactic component of 0.38, we find that only 0.01 remains for the portion of zodiacal light - i.e., practically nothing. If, on the other hand, it is assumed that $p = 0.87$, then we find a significantly larger value of 0.54.

An analogous conclusion is reached from a discussion of the observations on August 7 - 8, 1946. For $p = 0.85$, we have

$$C = 1.14 \quad \text{and} \quad j = 3.6.$$

The zodiacal component at the pole is ($J_3 = 1.63$)

$$J_3 - 1.14 \cdot 1.167 - 0.38 = -0.08 \sim 0.$$

For $p = 0.87$, we find

$$C = 0.56; j = 4.7.$$

The zodiacal component equals 0.57.

Such a large change depends on the fact that the observations employed pertain to large zenith distances. It is also apparent that an exact determination of the zodiacal component is only possible with an exact knowledge of atmospheric transmission. Particular attention must be given to this point. The best method employs exact photometric methods to compare the brightnesses of stars at large zenith distances with stars close to the zenith. Even visual observations of the bright stars as they rise and set, employing the well-known method of Pickering, can provide satisfactory results, if the uniformity of the optical atmospheric properties are taken into account, especially in the horizontal direction.

/249

A very reliable method for determining p is the photoelectric measurement of solar radiation through a green filter in the early morning hours, when the atmosphere still retains its nocturnal transmission. Such measurements can easily provide an accuracy of up to 0.1%. Ye. V. Fesenkov also developed a method for observing the maximum of the solar corona, yielding results which closely corresponded to the classical method of Bouguer. The results obtained by this new method pertain to one moment in time, and therefore do not entail a condition of atmospheric transmission uniformity.

Due to the unreliability of the value for p , it is not possible to make a fully reliable determination of the zodiacal component around the pole. If we assume the value $p = 0.85$, then we find the following values for the zodiacal component in the region of the pole:

7 - 8 August		- 0.08
23 - 24 August		- 0.03
24 - 25 August		0.15
25 - 26 August		0.14
31, Aug. - 1, September		0.01
1 - 2 September		- 0.19
Mean		0.00

However, this result is not probable, since the observer (it can be said with certainty) is located in the medium itself of zodiacal matter, and therefore zodiacal light must be propagated throughout the celestial vault. Due to the lack of exact nocturnal determinations, one can start with the general characteristics of atmospheric transmission during the morning hours, based on the transmission indices determined with a halo photometer from August 24 to September 5, 1946. The values of p fluctuated from 0.85 to 0.87. If we assume 0.86 on the average, then the zodiacal component yields almost

/250

exactly 0.2 at the pole.

This value, which cannot be increased, directly contradicts the results of Van Rhijn (Ref. 76), who found that the zodiacal light comprises 43% of the total night sky brightness.

A knowledge of the zodiacal component at the celestial pole affords a very slight possibility of determining the absolute ionosphere airglow at any time during the night. If a photometer is used whose scale makes it possible to read off the brightness in the number of stars of the fifth magnitude per square degree, it is only necessary to measure the brightness of the sky close to the pole and to subtract 0.38, as well as the zodiacal component. The difference represents the ionosphere airglow J_0 , taking into account diffused troposphere light. The atmospheric component J at any other region in the sky will be

$$J = J_0 (p + 0.03)^{\sec z - \sec z_0} \sqrt{\frac{(1 + h)^3 - \sin^2 z_0}{(1 + h)^3 - \sin^2 z}}.$$

If this region is not too close to the galactic plane, a calculation may be made for it, or the total stellar brightness can be taken from the appropriate tables, followed by a correction for stellar light scattered in the troposphere.

As a result, a complete photometric analysis may be made of night airglow, which is applicable to any problem whatsoever.

CHAPTER 15

Photometric Observations of Zodiacal Light and Their Interpretation

In spite of its great brightness, zodiacal light is barely under- /251
stood even by astronomy specialists. This can be explained, no doubt,
by the fact that this phenomenon does not have any specific contours
and develops only slowly and gradually throughout the night. It is
most readily visible only for a brief period of time before sunrise or
immediately after twilight. Nevertheless, the attentive observer can
discover clear traces of zodiacal light at any time during the night
when there is no moon or any foreign lights. Thus, for example, at
the beginning of autumn during the early hours of the night, the western
portion of the sky appears just as bright as does the eastern portion.
However, a light spot can be distinguished in the north-western portion,
which appears to be slightly raised above the horizon and which gradually
moves to the north, following the motion of the sun. This is the so-
called Northern Zodiacal Light, which combines its eastern and western
cone. During the summer solstice, this northern zodiacal light is very
clear, and extends throughout the entire northern portion of the sky,
virtually up to the celestial pole.

In addition, an extensive zodiacal zone can be seen in the southern
portion of the sky - faint diffused luminosity, which extends along the
ecliptic and which can be readily observed at a certain distance from
the Milky Way. This phenomenon can always be seen under favorable con-
ditions, when the ecliptic passes fairly high above the horizon and /252
there is no interference of the Milky Way. If this is not the case,
the observer cannot distinguish any contrast with the surrounding sky
background.

Under the same conditions and at a point opposite the sun, one
can observe a turbid spot having an oval shape, the so-called counter-
glow. Photometric observations of the counter-glow are extremely diffi-
cult, since this phenomenon is very small. Sandig obtained more or less
successful photographs of it in Equatorial Africa using a fast Leica
camera (Ref. 77).

In my opinion, usually in the autumn months, when the point oppo-
site the sun is located at the Aries-Pisces, the counter-glow has a
much sharper outline, an oval form, is extended along the ecliptic,
and does not exceed $5 - 6^\circ$. On the other hand, during early spring
in another period which is favorable for observing this phenomenon, it
appears in the form of a spot which is barely distinguishable having

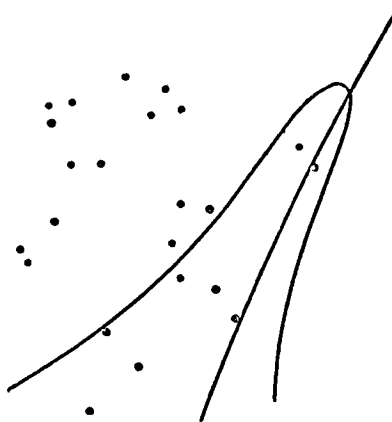
an indefinite circular form, whose diameter can be guessed - rather than determined - as being on the order of 10 - 15°.

After midnight, the sky begins to lighten considerably in the east, while the west remains dark as usual. This airglow has a diffused nature at first, and the impression is received that it encompasses the entire eastern horizon. This corresponds to the so-called diffuse light, according to the terminology employed by Serpieri (Ref. 78). However, after only 2 hours of the night in solar time, the outlines of an extensive cone of zodiacal light begin to stand out in the east; this light is inclined toward the horizon and expands and intensifies considerably as it approaches the sun. Shortly before dawn the brightness of the zodiacal light increases every minute. Finally, a narrow band of twilight appears, which spreads along the horizon, in contrast to the zodiacal light which rises sharply. Following this, the sky begins to lighten, and the approaching dawn conceals the outline of the zodiacal light and all the cosmic objects. /253

We would like to point out that the outer form of the zodiacal light, its "contour" which is usually drawn by observers, corresponds to the geometric location of points having maximum contrast. It appears that this phenomenon represents a system of isophotes which make it possible to determine the brightness J at any point in the sky. In this case, we can write the equation for the line of greatest contrast in the form

$$\frac{d^2 \lg J}{dn^2} = 0,$$

while the derivative is taken along the normal to the isophote. If the



October 4, 1945

Figure 16

Apparent Contour of Zodiacal
Light.

appropriate calculations are performed, it is then found that in the case of zodiacal light the apparent contour is much more elongated than the isophotes. Generally speaking, the form of this phenomenon depends on several factors - mainly on atmospheric absorption and on the superposition of the atmospheric and galactic components, which behave differently. The eye of the observer traces what appears to him to be the light patterns of the point of greatest contrast, and combines them into one contour. Figure 16 illustrates the outline of zodiacal light in the eastern sky on the night of October 3 - 4, 1945. The lines of greatest brightness are shifted considerably to the north with respect to the ecliptic. The zodiacal light is greatly elongated toward the north in its portions which are close to the horizon, but contracts upward. When the ecliptic is in a different position, the nature of the phenomenon changes. There is nothing remarkable in the fact that the apparent outlines of the zodiacal light are variable. Visual observations of this phenomenon, with which the scientific literature of the 19th and 20th Centuries abounds, cannot therefore be of any scientific significance.

/254

Zodiacal light must be studied by accurate methods of absolute photometry, which make it possible to express the observed brightnesses in a stellar scale. Observations must be corrected for the atmospheric and galactic components. A difficulty is encountered, however, in the uncertainty with which they can be determined. Our theory for the illumination of the night sky includes: h - height of the effective ionosphere layer; p - transmission index; $L(b)$ - total stellar brightness at different galactic latitudes; and finally, $\mu f(\theta)$ - the scattering indicatrix which determines the nature and intensity of additional airglow in the troposphere. The quantities h , p , $\mu f(\theta)$ apparently change from day to day. It is difficult to determine them for each night, and we must therefore confine ourselves to mean values. This is valid for h and $\mu f(\theta)$, but the transmission index p must be determined accurately, within at least 1%. We have a very inaccurate knowledge of the function $L(b)$. Actually, this function is determined by numerical integration of stellar brightness between the 7th and 20th stellar magnitudes on the basis of computations of the number of stars in each interval of photographic stellar magnitudes. In addition, the total stellar brightness must be extrapolated from the 20th magnitude up to infinity, and this extrapolation is absolutely unreliable for very low galactic latitudes. The disagreement between these calculations, based on data given by Seares and Van Rhijn, amounts to 100% of the Milky Way, with good agreement at a sufficient distance from it (Ref. 79). Therefore, in general it is not possible to determine the brightness of the galactic component in the Milky Way itself. The inaccuracy of $L(b)$ determined outside of the Milky Way depends on the necessity of introducing a correction for the color index, in order to change from photographic to photovisual total brightness. This represents a very

serious source of errors.

Thus, for example, according to my calculations for a galactic latitude of 15° the total stellar brightness from the 7th to the 20th magnitude is 0.54 in photographic rays, and in visual rays with the color index of Seares it equals 1.08 - i.e., exactly two times larger. It thus follows that a small error in the distribution of the color indices can change the quantity $L(b)$ to a significant extent. In addition, the nonuniformity of the stellar distribution with respect to longitude has a certain effect. /255

This nonuniformity is not of great importance. Actually, if the deviation from the mean distribution of a number of stars, according to data given by Seares (Ref. 80) is taken into account, the maximum difference in the total brightness is no more than 25%.

For example, the total brightness in photographic rays from the 7th to the 21st stellar magnitude in the galactic plane is 0.91 for the mean stellar distribution. If the same calculations are performed, taking into account the data given by Seares, for the region of the dark lane in the Aquila constellation, we find a decrease in the total brightness, equaling 0.23. Approximately the same proportion is retained in photovisual rays.

Due to this fact, observations of the zodiacal light close to the Milky Way cannot lead to reliable results. It is extremely desirable to determine in advance the integral brightness of the galaxy close to the Milky Way, when the influence of zodiacal light cannot be perceived.

The best method for observing the eastern or western zodiacal cone consists of drawing its photometric cross-sections at one and the same zenith distance, equaling approximately 75° and, in any case, not exceeding 80° . The observations are initiated beforehand, when the given region in the sky is located at a distance of about 90° from the sun with respect to the ecliptic. As the cone rises due to daily rotation, the brightness determinations of the sky are continued along this fixed almucantar, encompassing with respect to the azimuth not less than 150° in the region of the ecliptic through each 10° , and at a large distance from it through 20° and 30° . As a result, several zodiacal light cross-sections are obtained, which correspond to the same values of the atmospheric component. At the beginning and at the termination of each series, the sky brightness is determined around Polaris, which serves to separate the absolute value of atmospheric airglow and to control its uniformity during the night. However, in order to accelerate the operation one must usually take cross-sections which are higher with respect to the almucantar, gradually lowering them to the horizon, and the work must be carried out under this method only in the last hours before dawn. The /256

evening zodiacal cone can be observed in reverse order.

Such an observational method provides a great savings in processing time. Actually, in order to calculate the corrections given above it is necessary to compute the coordinates in all the systems for each point in the sky at which the total brightness is observed - namely, altitude and azimuth, hour angle, right ascension and declination, celestial longitude and latitude. The final values of the zodiacal light brightness, which do not include the atmospheric and galactic components, pertain to the coordinate system as a function of the celestial latitude and difference in longitude with respect to the sun. Isophotes are drawn along the plotted points, and thus the actual form of the phenomenon can be determined. By way of an example, we shall present the reduction of observations during several days in 1945 and 1946.

The mean brightness of the sky around Polaris was as follows.

3 - 4 Oct., 1945	1.43	15 - 16 Oct., 1945	1.53
4 - 5 "	1.49	1 Sept., 1946	1.55
6 - 7 "	1.46	3 "	1.45
7 - 8 "	1.49	5 "	1.60
9 - 10 "	1.53	6 "	1.64
14 - 15 "	1.46	8 "	1.42

Calculating the stellar component with a correction for the light scattered in the troposphere, we can find the ionosphere airglow C_0 close to the pole.

Multiplying the quantity C_0 by the factor

/257

$$F(z, p) = (p + 0.03)^{\sec z - \sec z_0} \sqrt{\frac{(1 + h)^2 - \sin^2 z_0}{(1 + h)^2 - \sin^2 z}},$$

we can find the ionosphere component for any zenith distance, in particular for all z at which the zodiacal light was observed. In 1945, the transmission index on a plain after a long drought was assumed to be 0.80, on the average. In 1946, under mountainous conditions after rains had passed over, the transmission was considerably larger and equalled 0.85 - 0.86.

The difference in the factor F as a function of p is apparent only at large zenith distances. Thus, for example, for values of p equaling 0.819 and 0.861 we have the following values of F :

TABLE 36

z	45°	50°	55°	60°	65°	70°	75°	80°	p
$F \dots$	0.93	1.04	1.08	1.17	1.25	1.29	1.29	1.10	0.819
$F' \dots$	0.98	1.04	1.06	1.14	1.19	1.21	1.14	0.89	0.861

After introducing a correction for the ionosphere, we introduce another correction for the total stellar brightness. As was indicated above, this correction is assumed in the form

$$L(b)p^{\sec z} + 0.355 \overline{\varphi(z,z)} \sec z \left(1 - \frac{1}{4} \sin^2 b \right).$$

The first term designates the total stellar brightness caused by atmospheric absorption; the second term designates the correction for scattering of the same light in the troposphere. Auxilliary tables for determining the latter term for different p were given above. When these tables were derived, a stellar distribution was assumed which depended only on the galactic latitude and which was the same over all longitudes. In addition, it was assumed that the galactic plane passed through the zenith. This term always remains small, with the exception /258

TABLE 37

$z \backslash b$	0°	10°	20°	30°	40°	50°	60°	70°	80°	90°
0°	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.03	0.03
0°	0.05	0.05	0.05	0.04	0.04	0.04	0.04	0.04	0.04	0.04
20°	0.05	0.05	0.05	0.05	0.05	0.04	0.04	0.04	0.04	0.04
30°	0.05	0.05	0.05	0.05	0.05	0.05	0.04	0.04	0.04	0.04
40°	0.06	0.06	0.06	0.06	0.05	0.05	0.05	0.05	0.05	0.04
50°	0.07	0.07	0.07	0.06	0.06	0.06	0.05	0.05	0.05	0.05
60°	0.08	0.08	0.08	0.08	0.07	0.07	0.07	0.06	0.06	0.06
70°	0.11	0.11	0.11	0.11	0.10	0.10	0.09	0.09	0.09	0.08
80°	0.18	0.18	0.18	0.17	0.16	0.16	0.15	0.14	0.14	0.14

of only very large zenith distances, at which the observations have no meaning. Thus, for example, for $p = 0.85$ we have the values in Table 37. The final reduction for the total stellar brightness, taking into account both terms in the expression given above, is as follows for $p = 0.85$.

It is interesting to note that at large galactic latitudes a decrease in the first term is almost exactly compensated by an increase in the second, so that on the whole the galactic correction remains

TABLE 38

$z \backslash b$	0°	10°	20°	30°	40°	50°	60°	70°	80°	90°
0°	1.68	1.21	0.85	0.58	0.41	0.30	0.24	0.23	0.21	0.21
10°	1.67	1.22	0.86	0.58	0.40	0.29	0.24	0.23	0.21	0.22
20°	1.66	1.21	0.85	0.59	0.41	0.29	0.24	0.23	0.21	0.22
30°	1.64	1.19	0.84	0.58	0.41	0.30	0.24	0.22	0.22	0.21
40°	1.59	1.16	0.82	0.57	0.39	0.29	0.24	0.23	0.22	0.21
50°	1.56	1.14	0.81	0.56	0.39	0.29	0.24	0.22	0.22	0.22
55°	1.53	1.12	0.80	0.55	0.39	0.29	0.24	0.23	0.21	0.21
60°	1.47	1.08	0.77	0.54	0.38	0.29	0.26	0.23	0.21	0.21
65°	1.41	1.04	0.75	0.53	0.38	0.28	0.26	0.23	0.21	0.21
70°	1.31	0.97	0.70	0.51	0.37	0.29	0.25	0.23	0.22	0.21
75°	1.17	0.88	0.65	0.48	0.35	0.28	0.24	0.23	0.22	0.22
80°	0.95	0.74	0.56	0.43	0.33	0.28	0.24	0.23	0.22	0.22

/258

unchanged. On the other hand, close to the galactic latitude this correction is considerable, and is therefore unreliable. Local absorption in individual cosmic clouds can considerably change its magnitude. By way of an example, let us consider the region in the sky around Polaris. According to our Table, the galactic reduction for $z = 47^\circ$, $b = 30^\circ$ comprises 0.56, while 0.38 should be assumed. This difference is due to the presence of considerable light absorption in cosmic clouds blanketing the celestial pole region. For the morning cone of zodiacal light, located close to the winter Milky Way, which is fainter than the southern one, the values given in this table are in all probability too large for small galactic latitudes. However, as was indicated above, cosmic absorption is barely able to decrease the first term by more than 25%. Making such a reduction, we obtain the following values of the galactic reduction for several zenith distances.

/259

$$\frac{3}{4} L(b) p^{\sec z} + 0.355 \overline{\varphi(z, z_1)} \sec z \left(1 - \frac{1}{4} \sin^2 b\right).$$

For large galactic latitudes, the correction remains almost unchanged. Thus, the brightness of zodiacal light can be reliably derived only at a sufficient distance from the Milky Way. In this respect, October

TABLE 39

$z \backslash b$	0°	10°	20°	30°	40°	50°	60°	70°	80°	90°
70°	1.01	0.75	0.55	0.41	0.30	0.24	0.20	0.19	0.19	0.18
75°	0.91	0.69	0.52	0.39	0.30	0.24	0.21	0.20	0.19	0.19
80°	0.76	0.60	0.46	0.37	0.29	0.25	0.23	0.21	0.20	0.20

observations of zodiacal light in 1945 are more reliable.

/260

It was shown above that the apparent brightness of zodiacal light around the pole can be assumed to be 0.2 at the maximum. Therefore, the ionosphere component at this point is

$$C_0 = J_3 - 0.38 - 0.20,$$

where J_3 is the total observed brightness around the pole. If we know C_0 , we can find the ionosphere component at any other point in the sky. This quantity, as well as the galactic reduction at the same point, must be subtracted from the observed brightness of zodiacal light. The remainder represents the apparent brightness of zodiacal light, which must still be corrected for atmospheric absorption.

The latter operation entails significant difficulties. There is no basis for applying the general formula of Bouguer to such an extended object as zodiacal light. On the other hand, a rigorous calculation of troposphere scattering in the given case is impossible, since it requires a knowledge of the entire isophote system, which must still be determined.

This problem is quite complex and merits a detailed analysis. At the present, we shall confine ourselves to only approximative considerations. First of all, it can be stated that the brightness of the sky around the pole does not evidence a systematic increase as the zodiacal light rises. On the contrary, this quantity remains unchanged almost up to the beginning of dawn. This shows that the troposphere component of zodiacal light is, in any case, small for all of its bright portions. On the other hand, if zodiacal light were to cover the entire sky uniformly, then the correction for the troposphere would have the following form:

$$p^{\sec z}(1 + K),$$

which is equivalent to a significant increase in p . In addition, if the brightness in it increased toward the horizon independently of the azimuth - similarly to an increase in the ionosphere brightness - /265 then the desired correction would be $(p + 0.03)^{\sec z}$. In our case, this correction must differ to a lesser extent from the customary formula of Bouguer. Due to this fact, we can correct the apparent brightnesses obtained by the factor $p_1^{\sec z}$, assuming that $p_1 = 0.82$ in 1945 and $p_1 = 0.87$ in 1946 - i.e., by 0.02 more than the assumed transmission index. The somewhat arbitrary nature of this reduction is mitigated by the fact that the observations in the largest portion were carried out at the same almucantar.

By way of an example, let us present the observations in reduced form for October 9 - 10, 1945, and September 6, 1946. The first line

TABLE 40

9-10 Oct. 1945								
<i>J</i>	2.34	2.04	1.76	1.98	2.51	2.65	2.23	1.93
<i>l</i> _☉ - <i>l</i>	84	79.3	74	121	79	69	62	57
<i>β</i>	+ 3	13	29	-42.5	-14.5	+1	18	36
<i>z</i>	55	55	55	67	67	67	67	67
<i>b</i>	23	32	40	30	18	35	48	57
gal. red . .	0.69	0.49	0.37	0.48	0.73	0.42	0.29	0.24
atm. . . .	0.95	0.95	0.95	1.09	1.09	1.09	1.09	1.09
<i>J</i> _{app} . . .	0.70	0.60	0.44	0.41	0.69	1.06	0.85	0.60
<i>J</i> _v	0.95	0.81	0.60	0.64	1.08	1.65	1.32	0.94
<i>J</i>	2.10	2.42	2.84	2.90	2.41	2.28		
<i>l</i> _☉ - <i>l</i>	83	69	60	59	54	50		
<i>β</i>	-49.	-18.5	-1.5	1.5	17	26		
<i>z</i>	74	74	74	74	74	74		
<i>b</i>	25	22	40	43	57	62		
gal. red . .	0.52	0.57	0.33	0.31	0.23	0.22		
atm. . . .	1.05	1.05	1.05	1.05	1.05	1.05		
<i>J</i> _{app} . . .	0.53	0.80	1.46	1.54	1.13	1.01		
<i>J</i> _v	1.00	1.49	2.73	2.88	2.11	1.88		
<i>J</i>	1.92	2.20	2.22	3.00	2.53	2.14	2.30	
<i>l</i> _☉ - <i>l</i>	122	67	61	53	45	33	28	
<i>β</i>	-56	-27.5	-19.5	-1.5	17	36	54	
<i>z</i>	78	78	78	78	78	78	78	
<i>b</i>	39	17	27	45	62	72	63	
gal. red . .	0.32	0.58	0.44	0.29	0.22	0.20	0.22	
atm. . . .	1.02	1.02	1.02	1.02	1.02	1.02	1.02	
<i>J</i> _{app} . . .	0.58	0.60	0.76	1.69	1.29	0.92	1.06	
<i>J</i> _v	1.32	1.36	1.72	3.83	2.93	2.08	2.40	
<i>J</i>	2.05	2.32	2.90	2.88	2.39	2.09	2.18	
<i>l</i> _☉ - <i>l</i>	118	58	50	50	42	35	26	
<i>β</i>	-56	-18	0	0	18.5	37	55	
<i>z</i>	78	78	78	78	78	78	78	
<i>b</i>	29	30	18	48	66	74	63	
gal. red . .	0.41	0.40	0.27	0.27	0.22	0.21	0.22	
atm. . . .	1.02	1.02	1.02	1.02	1.02	1.02	1.02	
<i>J</i> _{app} . . .	0.62	0.90	1.61	1.59	1.15	0.86	0.94	
<i>J</i> _v	1.40	2.04	3.65	3.60	2.60	1.95	2.13	

/261

gives the observed brightnesses obtained directly by reading the photometer wedge. Then follow the coordinates of the observational points - longitude with respect to the sun, celestial latitude, zenith distance, galactic latitude. Corrections are given in accordance with these coordinates - gal. red. for the total stellar brightness; "atm." for the ionosphere; and the apparent brightness *J*_{app} is obtained, according to which the actual brightness of zodiacal light outside of the atmosphere *J*_v is finally obtained (see Table 40).

/265

/266

The values of *J*_v are plotted on the graph along the ecliptic coordinates, and isophotes are drawn along them which correspond to the

Table 40 (continuation)

9-10 Oct. 1945							
J	2.25	2.30	3.10	2.42	2.16	2.28	
$l_{\odot}-l$. . .	60	54	47	39	32	23.5	
β	-25.5	-17	11	20	38.5	57	
z	78	78	78	78	73	78	
b	23	33	52	70	75	61	
gal. red . .	0.49	0.37	0.25	0.21	0.20	0.22	
atm. . . .	1.02	1.02	1.02	1.02	1.02	1.02	
J_{app} . . .	0.74	0.91	1.84	1.19	0.94	1.04	
J_v	1.68	2.07	4.17	2.70	2.13	2.36	
J	2.23	2.40	3.40	2.94	2.57	2.07	
$l_{\odot}-l$. . .	55	50	44	43	36	29	
β	-24.5	-16	-2	2	21.5	40	
z	78	78	78	78	78	78	
b	27	36	50	55	73	75	
gal. red . .	0.44	0.35	0.26	0.24	0.21	0.20	
atm. . . .	1.02	1.02	1.02	1.02	1.02	1.02	
J_{app} . . .	0.77	1.03	2.12	1.68	1.34	0.85	
J_v	1.74	2.33	4.8	3.80	3.04	1.93	
J	2.09	2.27	2.66	3.29	3.65	2.62	2.25
$l_{\odot}-l$. . .	119	57	47	42	40	33	26
β	-58	-32	-15	-1	3.5	2.3	41.5
z	78	78	78	78	78	78	78
b	30	19	38	53	58	76	75
gal. red . .	0.40	0.55	0.33	0.25	0.23	0.20	0.20
atm. . . .	1.02	1.02	1.02	1.02	1.02	1.02	1.02
J_{app} . . .	0.67	0.70	1.31	2.02	2.40	1.40	1.03
J_v	1.52	1.58	2.97	4.58	5.43	3.18	2.33
J	1.93	2.20	2.91	4.35	2.63	2.28	
$l_{\odot}-l$. . .	106	51	40	34	27	20	
β	-58	-29	-12.5	6	25	44	
z	78	78	78	78	78	78	
b	24	24	43	63	81	74	
gal. red . .	0.57	0.57	0.30	0.22	0.20	0.21	
atm. . . .	1.02	1.02	1.02	1.02	1.02	1.02	
J_{app} . . .	0.34	0.61	1.59	3.12	1.46	1.05	
J_v	0.77	1.38	3.60	7.07	3.31	2.33	

/262

integral brightness values in the number of stars of the fifth magnitude per square degree. Attention should be called to the extensive form of the different isophotes, which are in sharp contrast to the apparent contour of the phenomenon. /266

Let us first examine the brightness change in zodiacal light along the ecliptic. Let us read off the angular distances from the sun of the different isophotes, which are given in Table 41.

It must first be stated that the absolute brightness of zodiacal /267

Table 40 (continuation)

<i>J</i>	2.34	2.20	2.52	3.26	4.10	3.49	2.52		
<i>l</i> _☉ - <i>l</i>	47	44	42	37	31	27	20		
<i>β</i>	-28	-24	-20	-11	7	17	36		
<i>z</i>	78	78	78	78	78	78	78		
<i>b</i>	21	27	32	37	47	66	77		
gal. red . . .	0.51	0.44	0.38	0.34	0.28	0.21	0.20		
atm.	1.02	1.02	1.02	1.02	1.02	1.02	1.02		
<i>J</i> _{app}	0.81	0.74	1.12	1.90	2.80	2.26	1.30		
<i>J</i> _v	1.84	1.68	2.54	4.30	6.35	5.12	2.94		
<i>J</i>	2.33	2.10	1.90						
<i>l</i> _☉ -	12	6	-14						
<i>β</i>	55	64	77						
<i>z</i>	78	78	78						
<i>b</i>	63	54	33						
gal. red . . .	0.22	0.25	0.37						
atm.	1.02	1.02	1.02						
<i>J</i> _{app}	1.09	0.83	0.51						
<i>J</i> _v	2.45	1.86	1.15						
6 Sept. 1946									
<i>J</i>	1.73	1.89	2.50	2.78	2.49	2.34	2.26	2.14	2.01
<i>l</i> _☉ - <i>l</i>	63.6	65.6	72.1	75.6	80.2	90.6	97.1	107.1	118.2
<i>β</i>	52.3	38.2	22.5	15.5	9.2	-3.0	-7.6	-11.6	-17.6
<i>z</i>	51.2	51.2	51.2	51.2	51.2	51.2	51.2	51.2	51.2
<i>b</i>	30	25	14	8	1	-14	-22	-30	-45
gal. red . . .	0.56	0.67	1.01	1.22	1.52	1.01	0.76	0.56	0.34
atm.	1.12	1.12	1.12	1.12	1.12	1.12	1.12	1.12	1.12
<i>J</i> _{app}	0.05	0.10	0.37	0.44	-0.15	0.21	0.38	0.46	0.35
<i>J</i> _v	0.06	0.12	0.46	0.55	—	0.26	0.47	0.57	0.43
<i>J</i>	2.22	2.25	2.70	2.96	3.71	3.10	2.78	2.53	1.98
<i>l</i> _☉ -	38.6	48.9	56.6	61.5	65.9	72.1	78.2	85.2	102.1
<i>β</i>	50	33.3	16.7	13.8	1.3	-6.3	-13	-19	-28.4
<i>z</i>	65	65	65	65	65	65	65	65	65
<i>b</i>	45.6	37°	24.6	17°	9°	0.5	-3.4	-17	-35
gal. red . . .	0.32	0.43	0.62	0.84	1.03	1.39	1.11	0.84	0.45
atm.	1.36	1.36	1.36	1.36	1.36	1.36	1.36	1.36	1.36
<i>J</i> _{app}	0.54	0.46	0.72	0.76	1.27	0.35	0.31	0.33	1.17
<i>J</i> _v	0.74	0.64	0.99	1.04	1.74	0.43	0.42	0.45	0.23

/263

light at the same angular distance from the sun in 1945 was approxi- /267
 mately 20% greater than in 1946. In view of the fact that the ob-
 servations were performed under completely different conditions and
 taking into account the indeterminacy of our reductions, this dif-
 ference cannot be regarded as completely realistic. The brightness
 distribution in both cases was almost exactly identical. Small
 deviations pertain only to the most far-removed and very weak portions
 of the phenomenon, which can be barely distinguished on the over-all
 sky background, and which are therefore observed with a large amount /268
 of relative error. The observed distribution can be compared with

Table 40 (continuation)

J	2.56	2.47	2.83	3.25	3.56	3.06	2.29		
$l_{\odot}-l$	12.6	27.6	38	46.6	56.6	67.6	82.6		
β	64.5	49.4	31.5	15	-1.7	-16.5	-29.4		
z	70.8	70.8	70.8	70.8	70.8	70.8	70.8		
b	51.3	53	46	32	16	-1	-20		
gal. red	0.29	0.28	0.32	0.48	0.80	1.14	0.80		
atm.	1.45	1.45	1.45	1.45	1.45	1.45	1.45		
J_{app}	0.82	0.74	1.06	1.32	1.31	0.47	0.04		
J_v	1.24	1.12	1.61	2.00	2.00	0.72	0.06		
J	2.70	3.88	4.16	3.35	2.60				
$l_{\odot}-l$	20.1	31.2	40.2	49.8	60.6				
β	49.0	31.5	14.0	-2.6	-18.6				
z	74	74	74	74	74				
b	57.1	50.6	37.3	20.	3.00				
gal. red	0.26	0.29	0.44	0.65	1.11				
atm.	1.48	1.48	1.48	1.48	1.48				
J_{app}	0.96	2.11	2.24	1.22	0.01				
J_v	1.56	3.43	3.64	1.98	0.02				
J	2.14	2.41	2.86	2.97	2.29	3.49	2.39	2.29	
$l_{\odot}-l$	17.6	27.6	35	42.2	50.6	60.3	74.0	86.6	
β	69.4	53.7	35.6	18.4	2	-13.6	-27	-37	
z	67	67	67	67	67	67	67	67	
b	46.2	51.0	48.1	37.0	23.0	6	-11.5	-30	
gal. red	0.32	0.28	0.30	0.42	0.67	1.18	1.00	0.52	
atm.	1.39	1.39	1.39	1.39	1.39	1.39	1.39	1.39	
J_{app}	0.43	0.74	1.17	1.16	0.23	0.92	0.00	0.38	
J_v	0.61	1.04	1.65	1.64	0.32	1.30	0.00	0.54	
J	2.94	2.95	3.36	4.60	5.45	4.77	4.13	3.43	2.54
$l_{\odot}-l$	-5.9	11.6	22.6	31	35.4	33.6	44.6	50.6	64.6
β	65	50	32.4	14.5	5.4	-3	-11.4	-19.7	-34.1
z	76.9	26.9	76.9	76.9	76.9	76.9	76.9	76.9	76.9
b	54	61.5	59	45	37	29	20	10	-8
gal. red	0.26	0.24	0.24	0.32	0.36	0.48	0.62	0.83	0.88
atm.	1.41	1.41	1.41	1.41	1.41	1.41	1.41	1.41	1.41
J_{app}	1.24	1.27	1.63	2.84	3.65	2.85	2.07	1.16	0.22
J_v	2.31	2.37	3.14	5.30	6.80	5.31	3.86	2.16	0.41

/264

the theoretical distribution under the assumption that the density of meteoric matter in space changes in a manner which is inversely proportional to the distance from the sun, and also to the spherical scattering indicatrix. The form of the indicatrix, however, is of no significant importance at angular distances of $30^\circ - 70^\circ$ from the sun. Under these assumptions, the brightness of zodiacal light must change proportionally to $[1 - \cos(l - l_0)]^{-1}$, which is expressed by the continuous curve in the graphs, with the appropriate choice of the proportionality coefficient.

/268

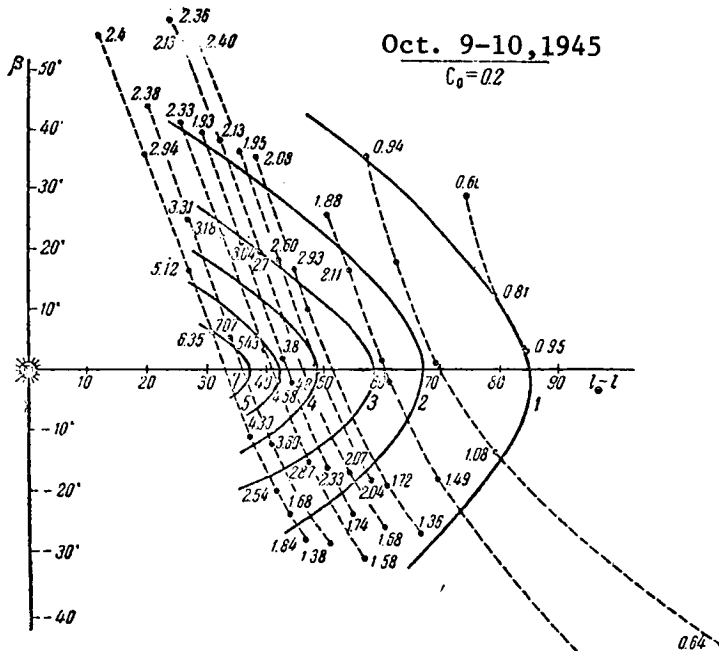
/269

As can be seen, the actual brightness changes somewhat more

Table 40 (continuation)

J	1.81	3.00	3.06	3.32	4.66	5.64	5.51	3.86	3.25
$l_{\odot}-l$. . .	-14.9	3.6	15.6	24.6	28.6	33.2	38	49.6	66.2
β	65	50.2	32.3	14.2	5.4	-3.1	-12	-33.6	-43.4
z	80	80	80	80	80	80	80	80	80
b	55	65	64	51	42	34	24	6	-13
gal. red . .	0.27	0.24	0.25	0.28	0.32	0.39	0.50	0.82	0.66
atm.	1.35	1.35	1.35	1.35	1.35	1.35	1.35	1.35	1.35
J_{app} . . .	0.19	1.41	1.46	1.69	2.99	3.90	3.66	1.70	1.24
J_v	0.41	3.03	3.19	3.68	6.50	8.50	7.96	3.70	2.70
J	3.15	3.67	5.26	6.41	6.34	4.31	3.17	2.48	
$l_{\odot}-l$. . .	2.6	14.6	23.6	27.6	32	36.6	48.0	64.9	
β	51	33	14.5	5.7	-3	-12	-33.4	-43.4	
z	80	80	80	80	80	80	80	80	
b	65	65	52	44	35	25	6	-12	
gal. red . .	0.24	0.24	0.28	0.31	0.38	0.50	0.82	0.72	
atm.	1.35	1.35	1.35	1.35	1.35	1.35	1.35	1.35	
J_{app} . . .	1.56	2.03	3.53	4.75	4.61	2.46	1.00	0.41	
J_v	3.40	4.51	7.9	10.4	10.0	5.35	2.17	0.89	

/265



/266

Figure 17

Isophotes of Zodiacal Light from Observations
on October 9 - 10, 1945.

rapidly than does the theoretical brightness, although the difference /269
is insignificant for small angular distances. From the aspect of the

TABLE 41

/267

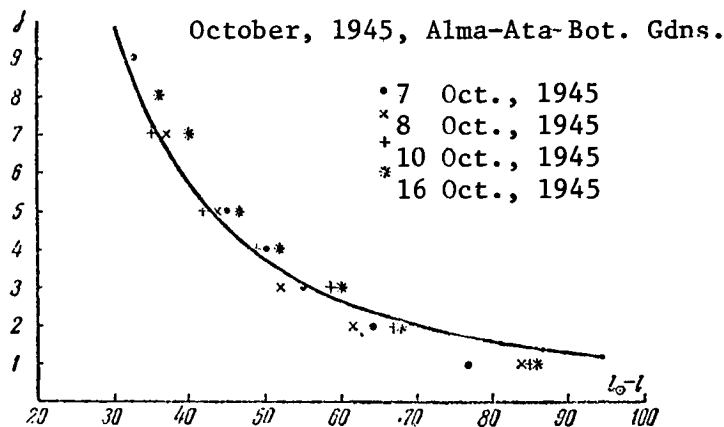
1945				
J _v	6-7 Oct.	7-8 Oct.	9-10 Oct.	15-16 Oct.
1	77°	84°	85°	86°
2	64	62	67	68
3	55	52	58.5	60
4	50	--	49	52
5	45	44	42	47
7	40	37	35	40
8	--	--	--	36
9	32-33	--	--	--
1946				
J _v	5 Sept.	6 Sept.	8 Sept.	
0.5	88°	87°	--	
1	74	69	70	
2	57	58	59	
3	--	50	48	
5	43	42	38	
7	--	35	--	
8	--	--	31	

theory for stationary distribution of meteoric matter, based on the assumption of gradual disintegration of asteroids (see Chapter 8), this difference can be explained by the addition of elliptical orbits (the precise law of inverse proportionality to the distance is valid only for circular orbits).

/269

A more serious problem arises in connection with the form of zodiacal light. As was shown above (Chapter 9), its isophotes can be represented theoretically on the basis of our hypothesis for the origin of zodiacal light, which leads to the following expression for spatial density

$$D = f(r)e^{-ksin\varphi}.$$



/268

Figure 18

Brightness of Zodiacal Light in the Ecliptic Plane as a Function of Angular Distance from the Sun in 1945.

The expression $f(r)$, which is identical for all planes, can be determined on the basis of only one stationary condition. It is interesting to note that the same expression $f(r)$ is obtained for any mechanism by which space is filled with cosmic matter when this mechanism occurs externally - whether it is the disruption of periodic comets captured by planets, the ejection of matter from asteroids, or finally the slow and gradual deposit of dust particles which are retarded by solar radiation.

/269

Not one of these processes can explain the distribution of matter outside of the ecliptic plane, without additional assumptions concerning the very essence of the meteoric matter origin.

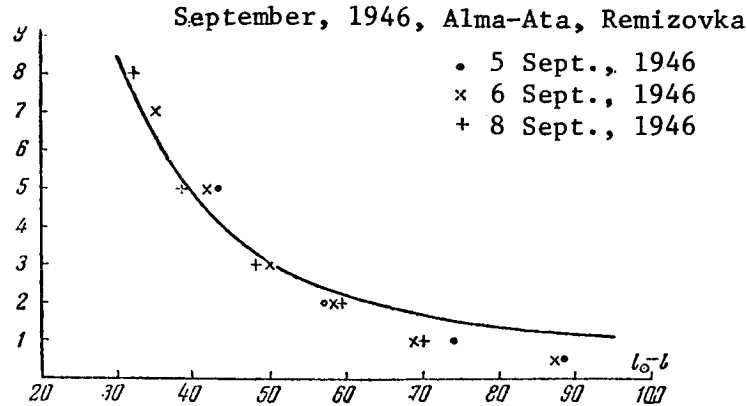
/270

It was shown above that the theory of asteroid disintegration makes it possible to construct zodiacal light isophotes (Chapter 9). However, these isophotes are too constricted, if it is assumed that dust leaves the asteroid surface without an initial velocity. It is interesting to note that the factor determining the density of zodiacal matter outside of the ecliptic

$$e = k \sin \varphi$$

(φ is the angle of inclination) is obtained from the known distribution of asteroid orbits, but the value of k is approximately 3 times smaller than is required in order to explain the zodiacal light isophotes.

This could indicate that particles leave the asteroid surface



/268

Figure 19

Brightness of Zodiacal Light in the
 Ecliptic Plane as a Function of Angular
 Distance from the Sun in 1946.

with a certain initial velocity, and also that the orbits of small asteroids, which comprise the basic material for meteoric matter, are inclined at rather large angles on the average. As was shown previously (Chapter 6), the region of the asteroid band must also abound in a fine dust, since under all pulverization conditions the greatest amount of matter must accumulate in this band. This explains the phenomenon of the zodiacal band. It can thus be assumed that bright cones of zodiacal light are constantly produced from this band, which is faint due to its distance from the sun and from the observer.

/270

In conclusion, we must point out certain discrepancies which must be clarified by future observations.

The Northern Zodiacal Light does not extend very far from the ecliptic and barely reaches the celestial pole, where the zodiacal component is very small, in any case. This small brightness is incompatible with the isophotes of the zodiacal light cones, which are obtained after introducing all of our reductions. The discrepancy is due to the fact that the zodiacal cones visibly extend too far to the north. This cannot be caused by a general increase in the sky airglow with latitude (latitudinal gradient) which, according to our results, is not of significant importance. The impression is gained that there is additional airglow close to the horizon, which is connected with the sun and which appears only when the sun drops slightly below the horizon about 20°, and even somewhat more. It has been generally acknowledged up to the present time, however,

/271

that twilight has completely terminated when the sun drops below the horizon by 16° , and in any case by 17° . This entails a serious problem of a theoretical and observational nature. It appears that photometric comparisons of the northern zodiacal light with both of its cones, followed by a spectroscopic investigation of them, can best serve to clarify this problem.

Moscow
January, 1947

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